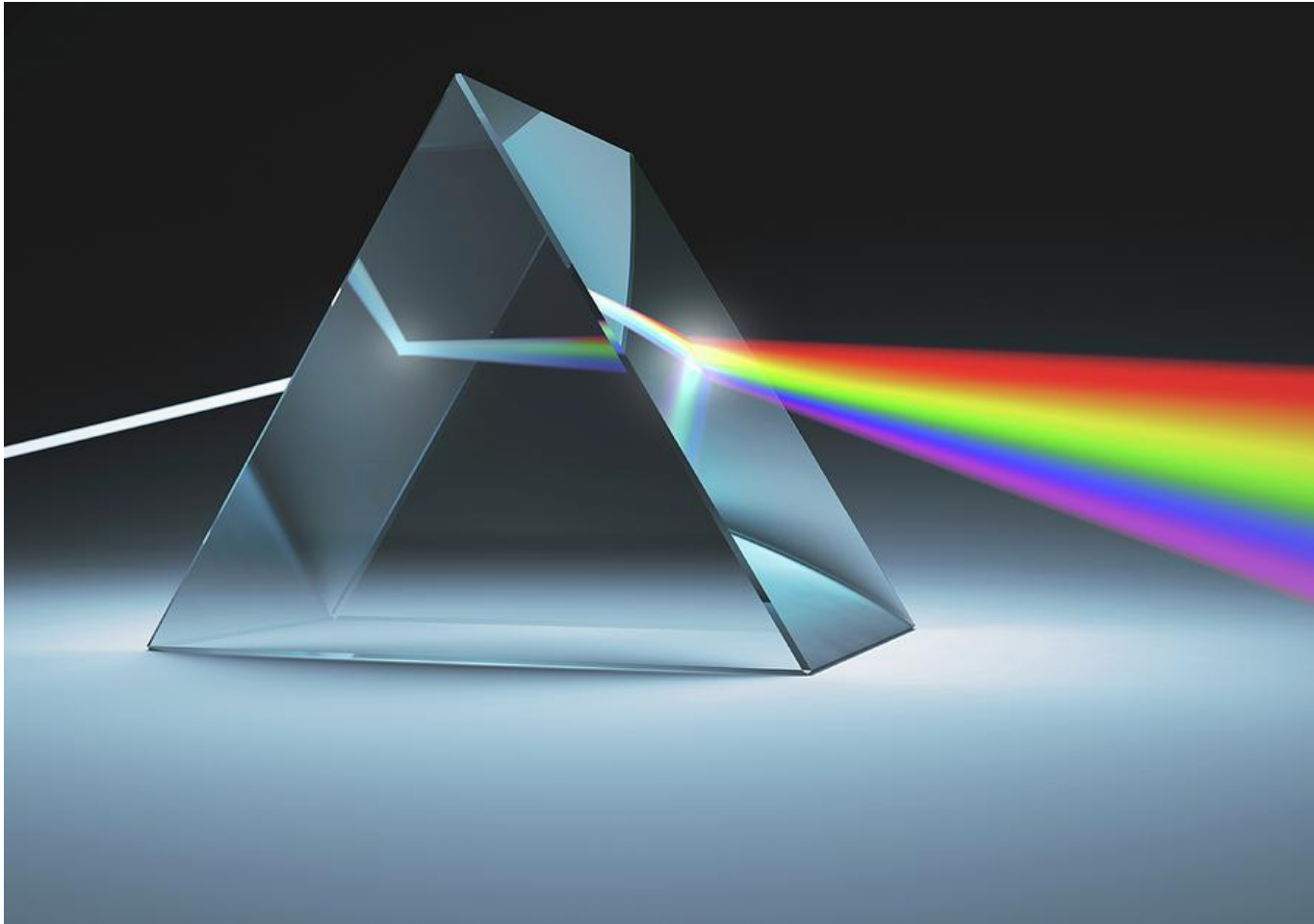
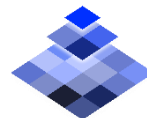


# Image formation



Yoni Chechik

[www.AlisMath.com](http://www.AlisMath.com)



# References

- <http://szeliski.org/Book/>
- <http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html>
- <http://www.cs.cmu.edu/~16385/>

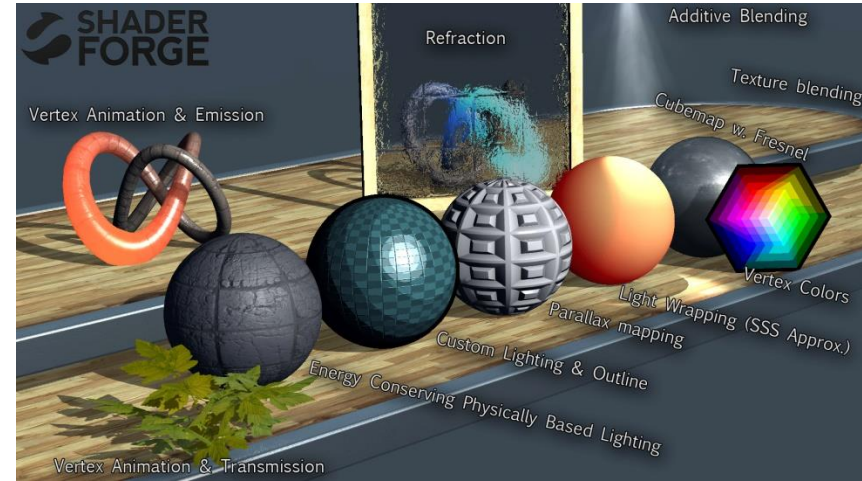
# Contents

- BRDF
- Pinhole camera
- Digital camera
- The human eye

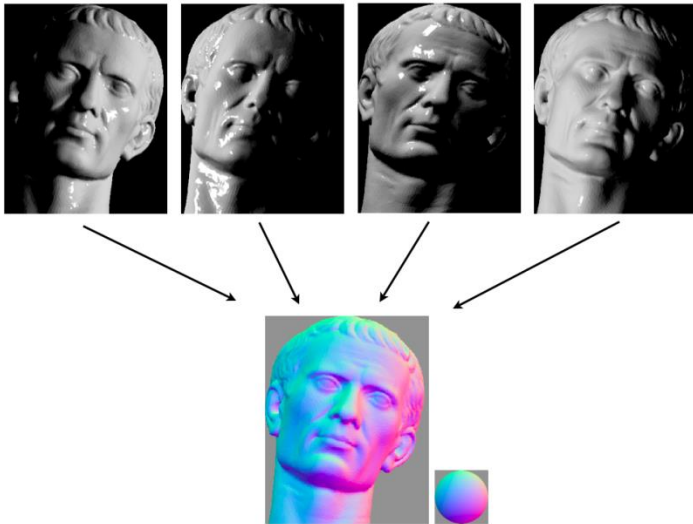
# Some motivation



Art  
(Exposure time)



3D game rendering  
(ray tracing)



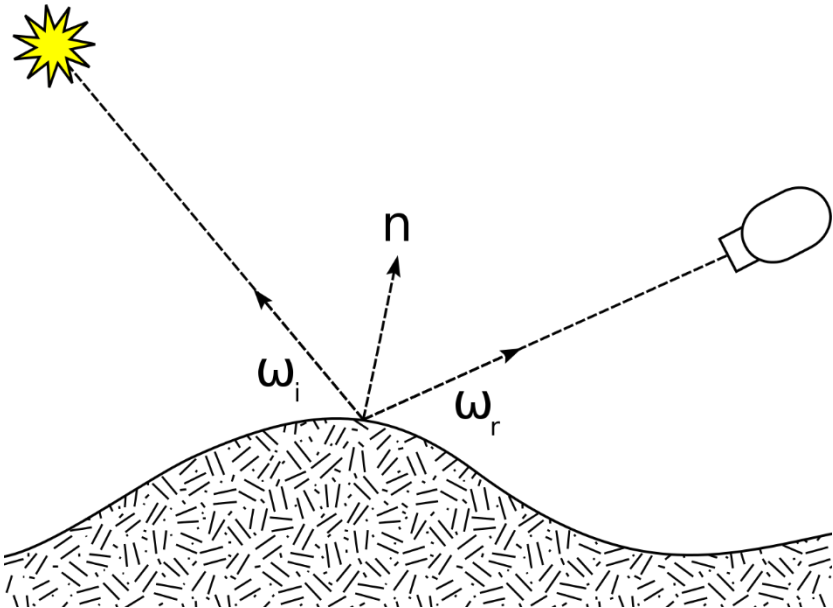
Computational photography  
(shape from shading- 3D reconstruction)



Computational photography  
(aperture blur shape)

# Image formation

- Image formation: what happens before the image is captured- how is it captured and why do we see it the way the we see it.



# Contents

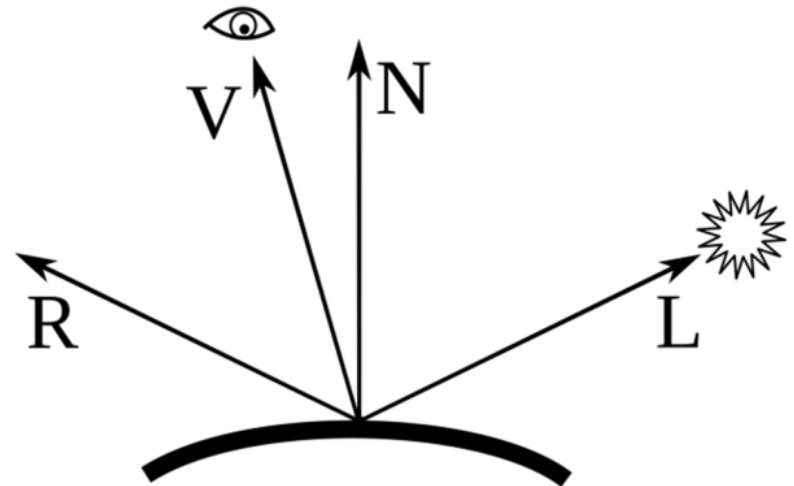
- **BRDF**
- Pinhole camera
- Digital camera
- The human eye

# BRDF

- **BRDF** or **bidirectional reflectance distribution function** is a function of the surface which is dependent on the input and output ray's direction relative to the surface normal:

$$\frac{I_r}{I_i} = f_{\hat{N}}(\hat{L}, \hat{V})$$

- The function output is basically a ratio of output reflected ray power ( $I_r$ ) to the input ray power ( $I_i$ ).
  - The complete (and more correct) definition is out of scope and can be [found here](#)]



# BRDF

- BRDF is used extensively in photorealistic rendering, and also in 3D reconstruction and image deblurring.
- When rendering 3D images with ray tracing, actual rays are generated and propagate through the scene using the BRDF of each material.





# BRDF of a diffused surface

- A perfectly diffused surface is a surface that light is reflected off it equally in all directions. The light intensity scattered off the surface is only determined by the input ray direction relative to the surface norm:

$$\frac{I_r}{I_i} = \kappa_d (\hat{L} \cdot \hat{N})$$

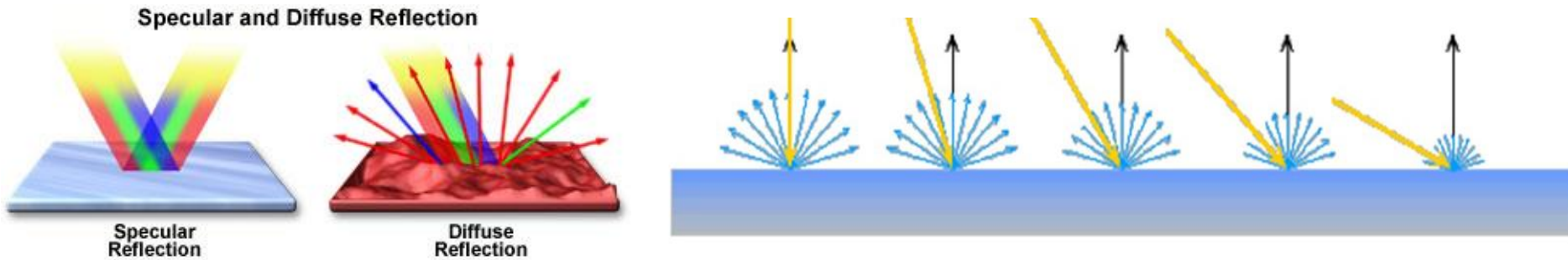


Figure 1

# BRDF of a diffused surface

$$\frac{I_r}{I_i} = \kappa_d (\hat{L} \cdot \hat{N})$$

- $\kappa_d \in [0,1]$  is the **diffuse reflectance coefficient** (sometimes named incorrectly **diffuse albedo**) which indicates the proportion of light reflected diffusely to the total input light intensity.
  - (some light is absorbed or reflected in a non-diffused way).
- This is known also as **Lambertian reflectance**.

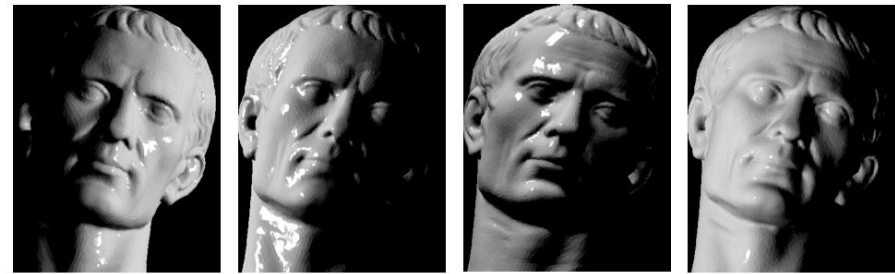
# BRDF of a diffused surface

- Diffused surface demo:

[http://learnwebgl.brown37.net/09\\_lights/lights\\_diffuse.html#a-webgl-demo-program-for-diffuse-lighting](http://learnwebgl.brown37.net/09_lights/lights_diffuse.html#a-webgl-demo-program-for-diffuse-lighting)

# Side note: photometric stereo

- Estimating surface normals from 2D images of different lighting conditions.
- Assumes Lambertian surface.
- Assumes infinite lighting plane (not a light point)- so all distances from the light source are the same intensities...



# Side note: photometric stereo

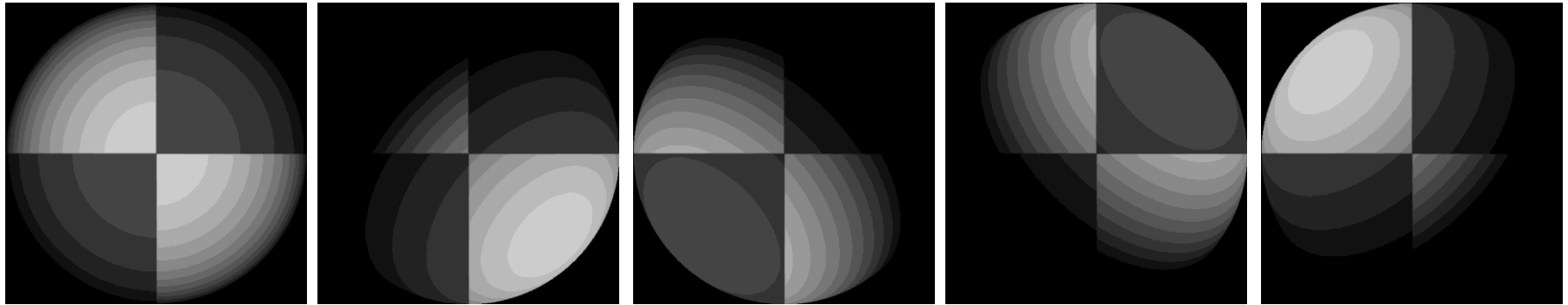
- Let's say we have  $m$  images of the same Lambertian object with different lighting scenarios. **Per pixel** we have:
  - $I_{m \times 1}$ :  $m$  different intensities of the same pixel- known.
  - $L_{m \times 3}$ :  $m$  different light vector directions - known.
  - $\alpha_{1 \times 1}$ : Light intensity- unknown.
  - $N_{3 \times 1}$ : the pixel's normal – unknown.
  - $\kappa_{d_{1 \times 1}}$ : diffuse reflectance coefficient- unknown.

$$I = \kappa_d \alpha L N = L G$$

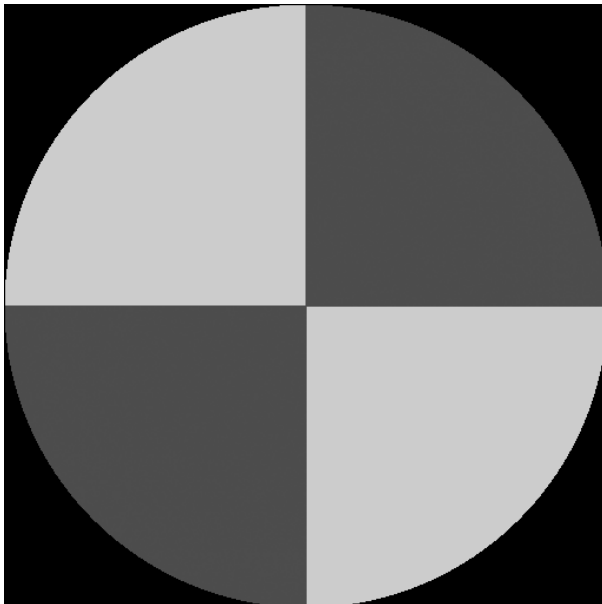
- From here, a simple psuadoinverse (taught in least squares class) can be applied to get  $N$  per pixel:

$$(L^T L)^{-1} L^T I = G$$
$$\|G\| = \alpha \kappa_d, \frac{G}{\|G\|} = N$$

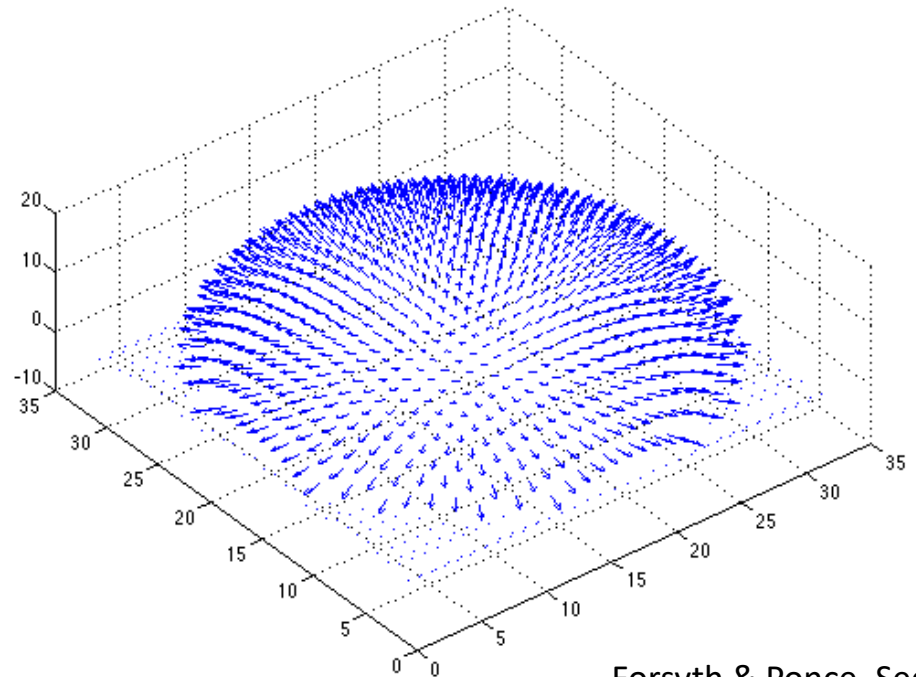
# Side note: photometric stereo



Recovered albedo

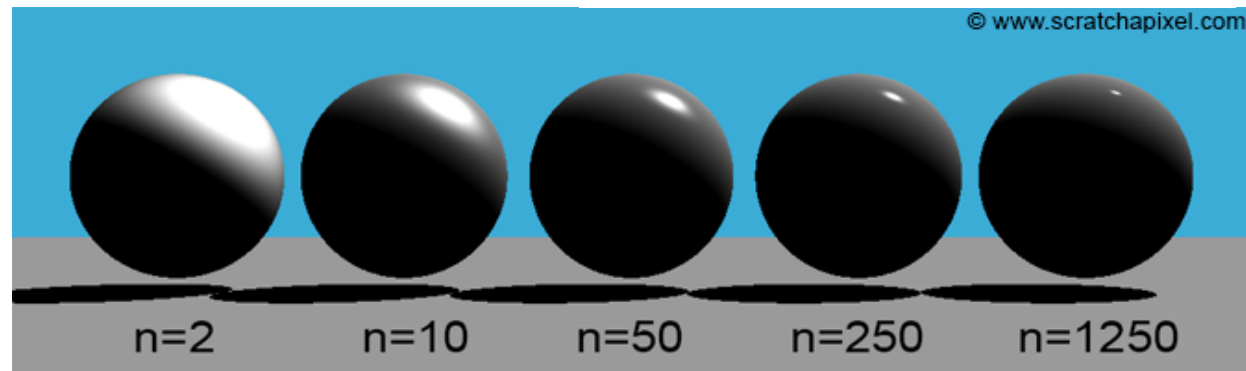
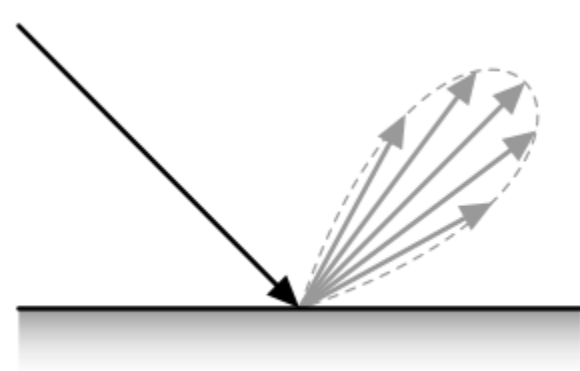
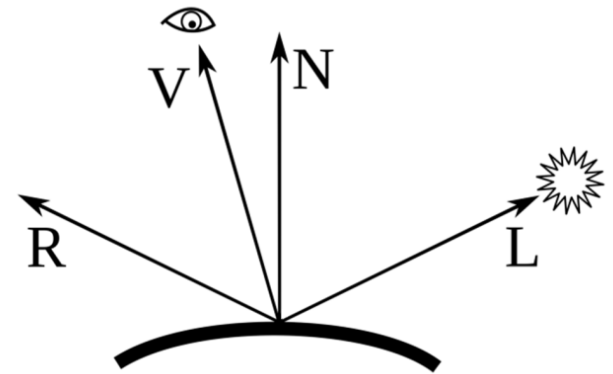


Recovered normal field



# BRDF of a specular surface

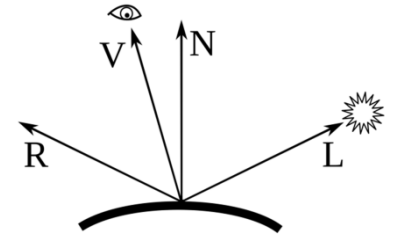
- A specular surface is a surface that the reflected light is returned mainly on the reflected vector of the lighting direction ( $\hat{L}$ ) relative to the normal. This direction is denoted as  $\hat{R}$ .
  - A mirror is a kind of specular surface where the ray is reflected **only** in the  $\hat{R}$  Direction.



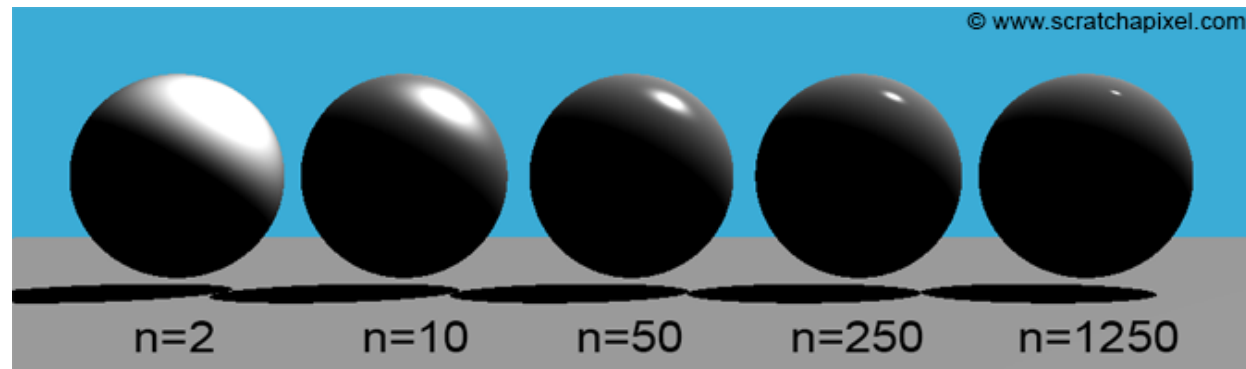
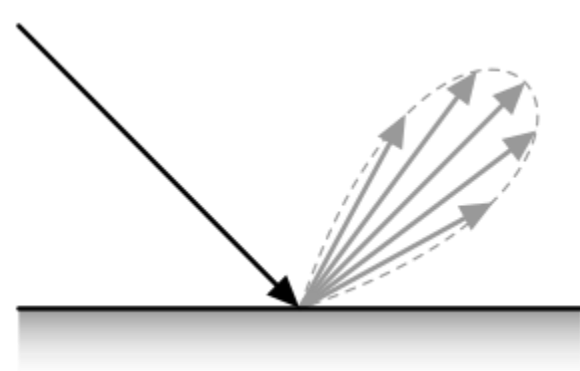
# BRDF of a specular surface

- A common model of a specular surface is:

$$\frac{I_r}{I_i} = \kappa_s (\hat{R} \cdot \hat{V})^n$$



- $n$  is a **shininess** constant for this material, which is larger for surfaces that are smoother and more mirror-like.
- $\kappa_s$  is the specular reflectance coefficient.  $\kappa_s \in [0, 1]$

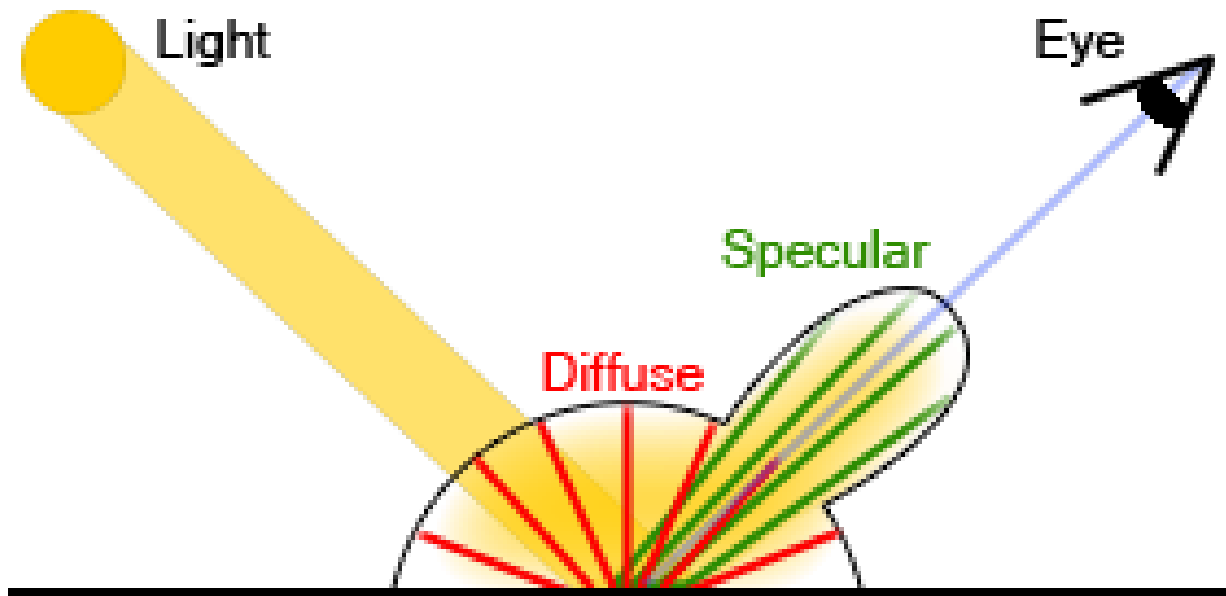




# Diffused and specular surface

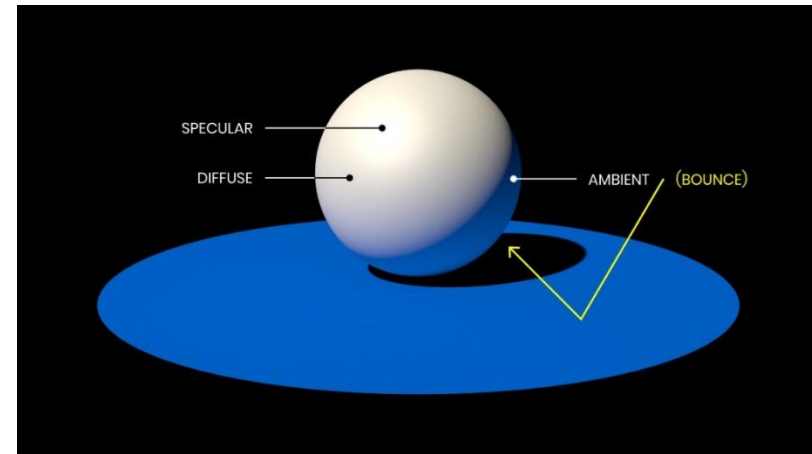
- Surfaces can combine both features with some kind of weighting such as:

$$\frac{I_r}{I_i} = \kappa_d (\hat{L} \cdot \hat{N}) + \kappa_s (\hat{R} \cdot \hat{V})^n$$



# Ambient light

- In some rendering techniques light reflection is only computed in first order- light ray **from source to object and to camera**.
  - It does not compute secondary reflection of light (= being reflected at one object and indirectly illuminates another object).
- To account for secondary reflection, a general light  $I_a$  is added that distributes homogeneously through the entire 3D scene, which is called **ambient light**

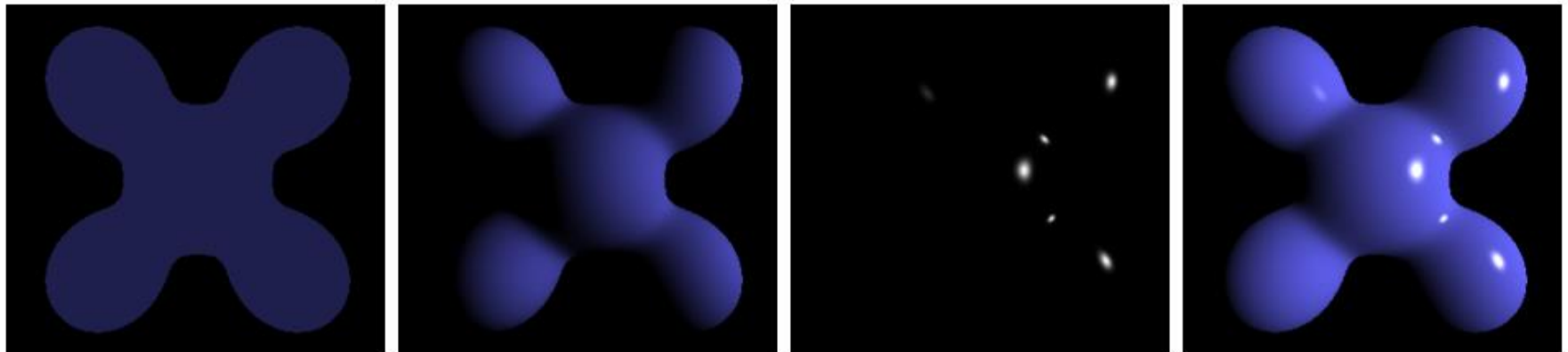


# Phong reflection model

- Bui Tuong Phong used ambient, diffused and specular reflections in a well known illumination model known as **Phong reflection model**, based on BRDF:

$$I_r = k_a I_a + \sum_{m \in \text{lights}} (k_d (\hat{L}_m \cdot \hat{N}) I_m + k_s (\hat{R}_m \cdot \hat{V})^n I_m)$$

- Demo: <http://www.cs.toronto.edu/~jacobson/phong-demo/> (change in dropdown to “Phong shading” ...)



Ambient

+

Diffuse

+

Specular

=

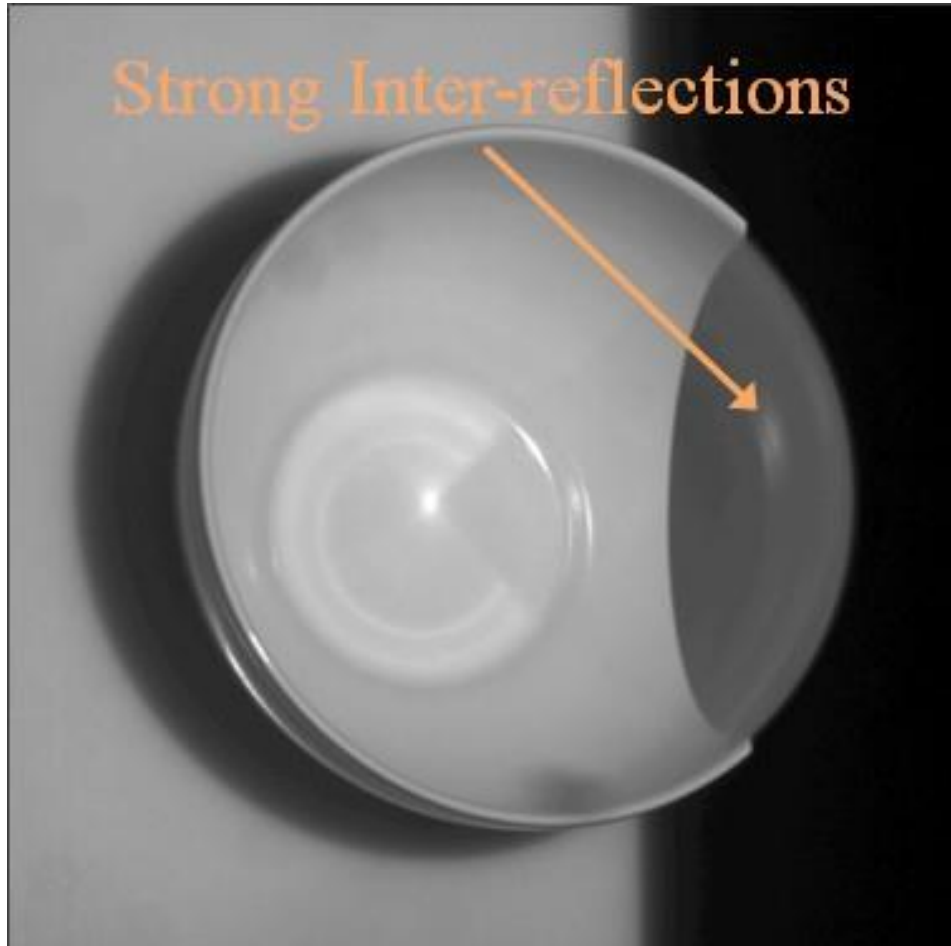
Phong Reflection

# Phong model limitations

- Phong model doesn't account for refraction and Interreflections.

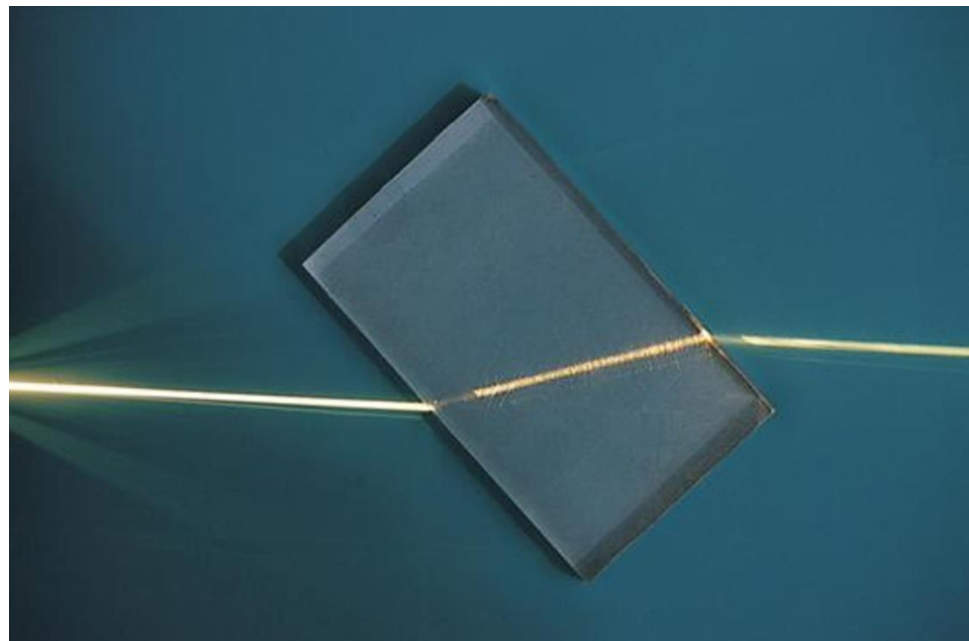
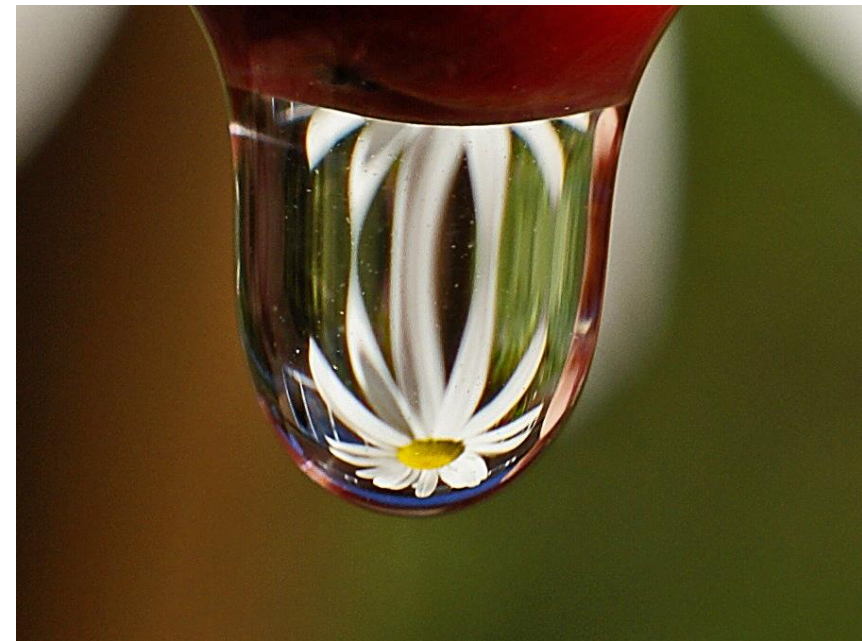
# Phong model limitations

- **interreflection** - reciprocal reflection between two reflecting surfaces



# BRDF limitations

- BRDF assumes single point of impact. It doesn't account for light rays that refract/scatters through a medium.
- **Refraction** is the change in direction of a wave passing from one medium to another or from a gradual change in the medium.
  - One point of entry, one (other) point of exit.



# BRDF limitations

- **Subsurface scattering** is a mechanism of light transport in which light that penetrates the surface of a translucent object, is scattered by interacting with the material, and exits the surface at a different point.
- One point of entry, multiple exit points.



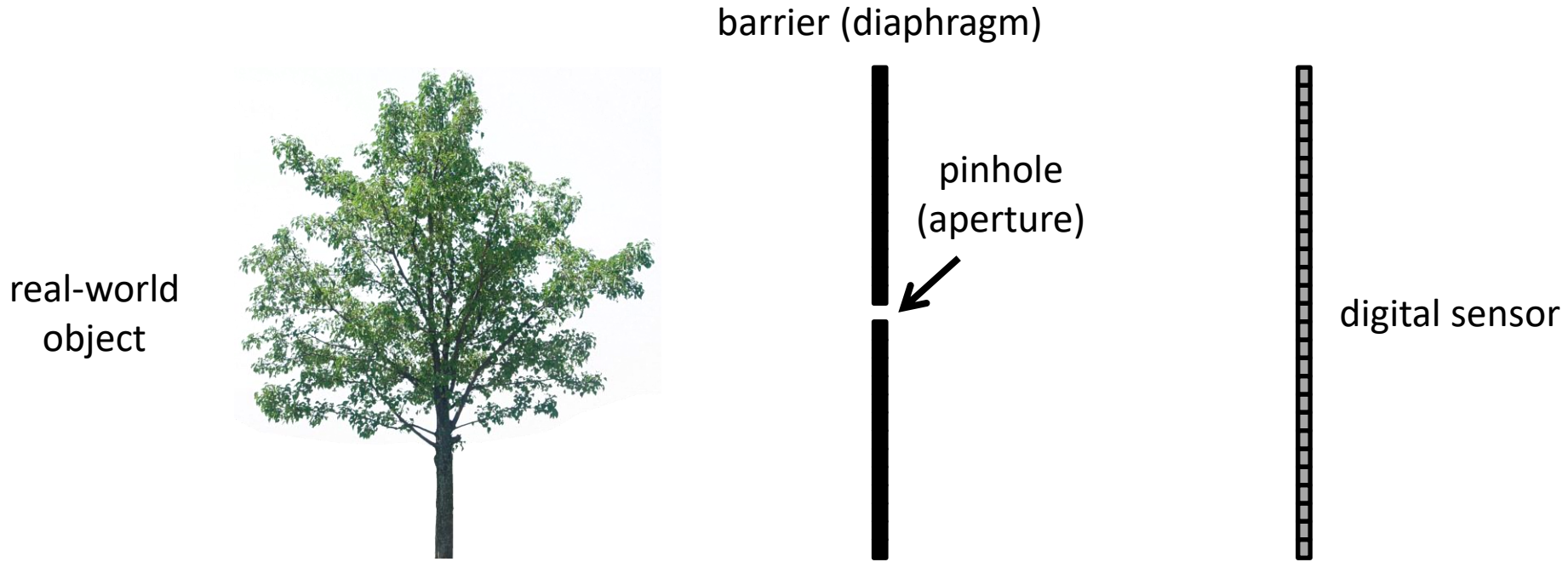
# Contents

- BRDF
- **Pinhole camera**
- Digital camera
- The human eye

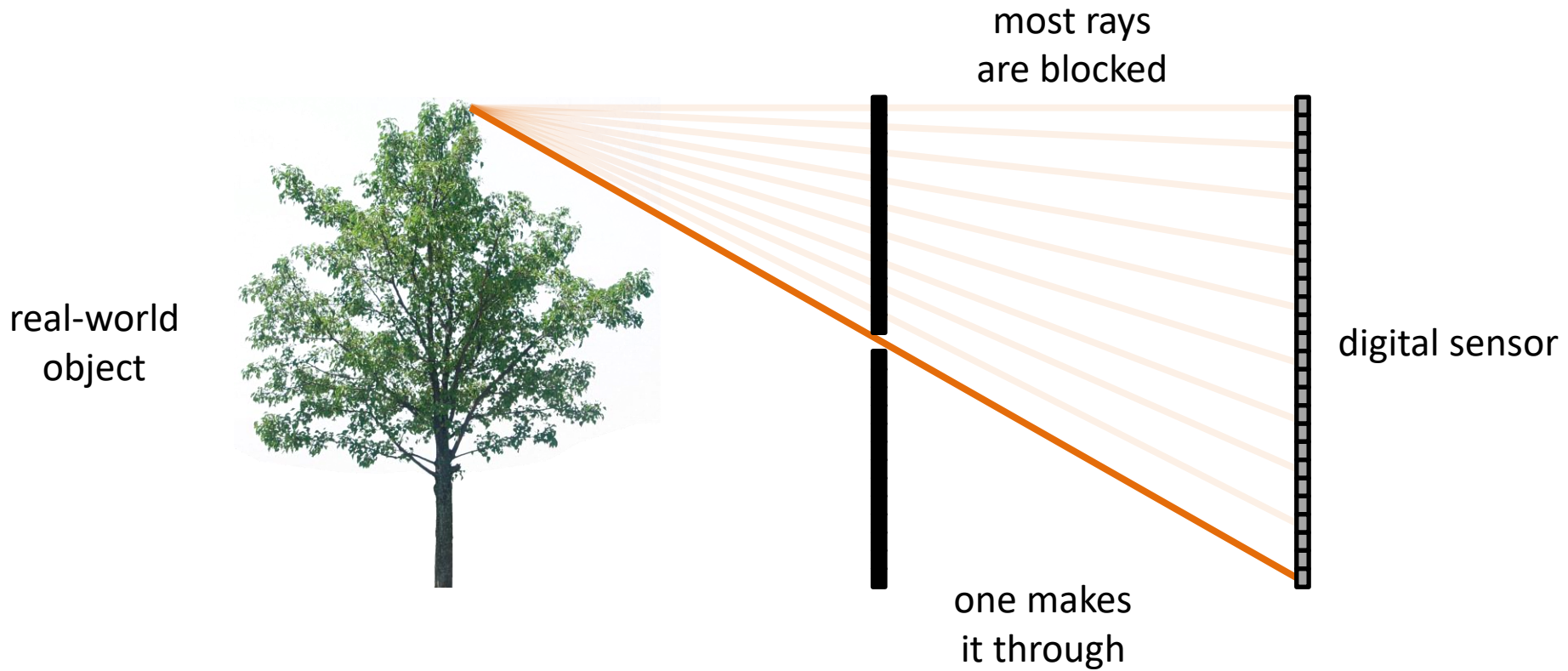


# Pinhole imaging

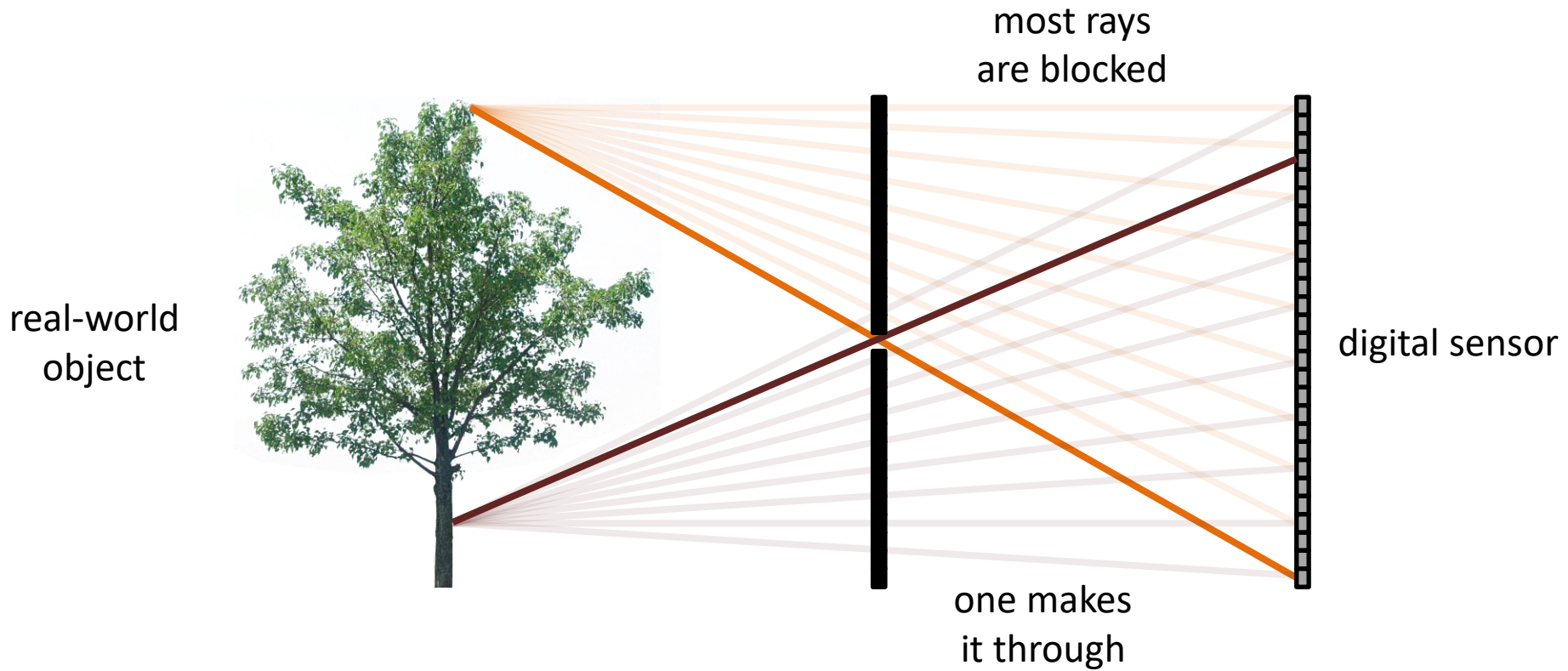
- What would an image taken like this look like?



# Pinhole imaging

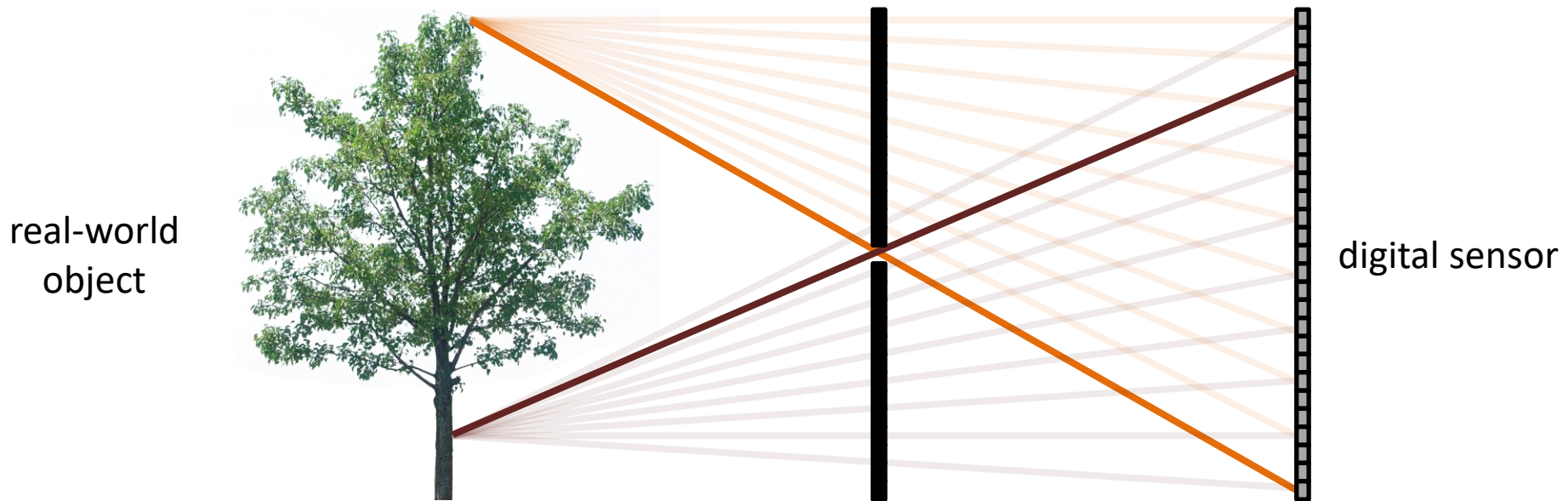


# Pinhole imaging



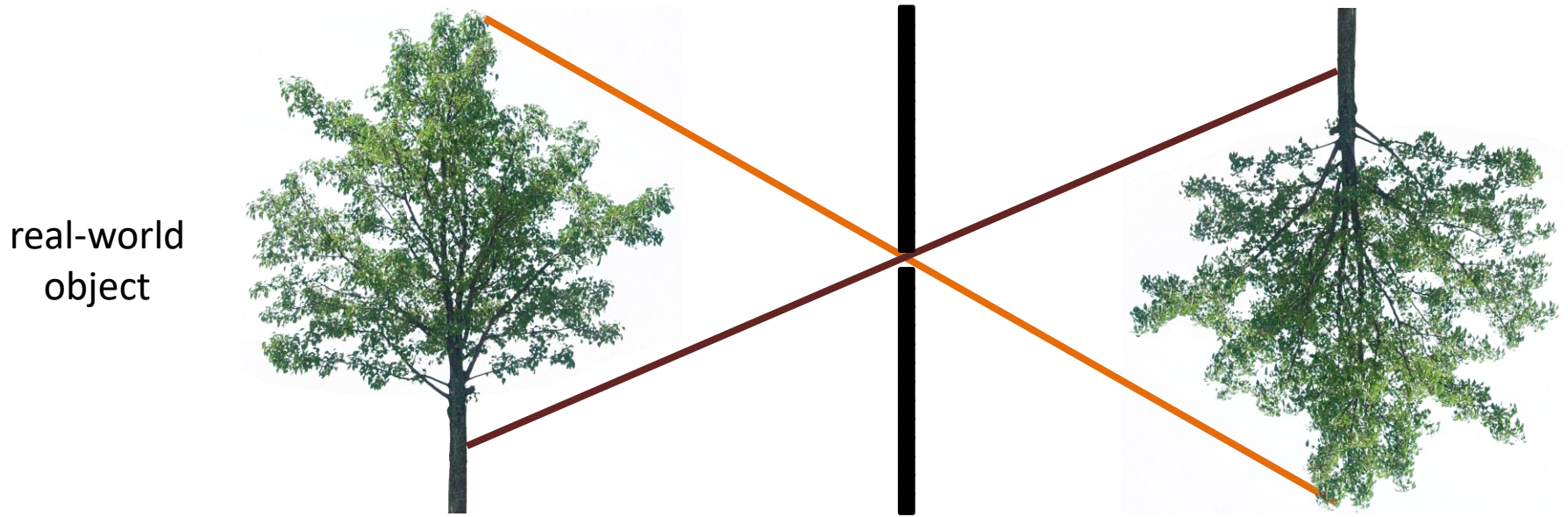
# Pinhole imaging

- Each scene point contributes to **only one** sensor pixel.



# Pinhole imaging

- Imaged object is inverted.





# Blur in pinhole cameras

- Since only one ray reaches every pixel in the pinhole camera, the image is always in focus.
  - (blur can come only from movement of camera or objects in time of exposure)



# Pinhole camera a.k.a. camera obscura

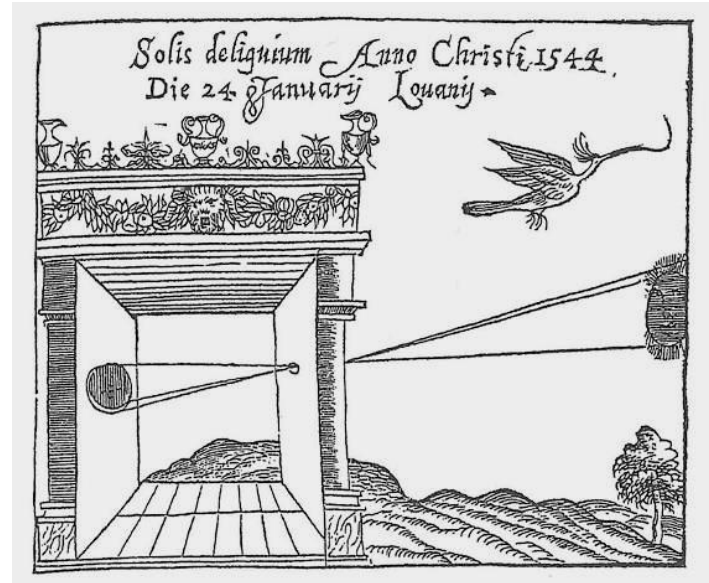
- **Camera obscura** - from Latin, meaning "dark room".

First mention ...



Chinese philosopher Mozi  
(470 to 390 BC)

First camera ...



Greek philosopher Aristotle  
(384 to 322 BC)

# Examples of camera obscura



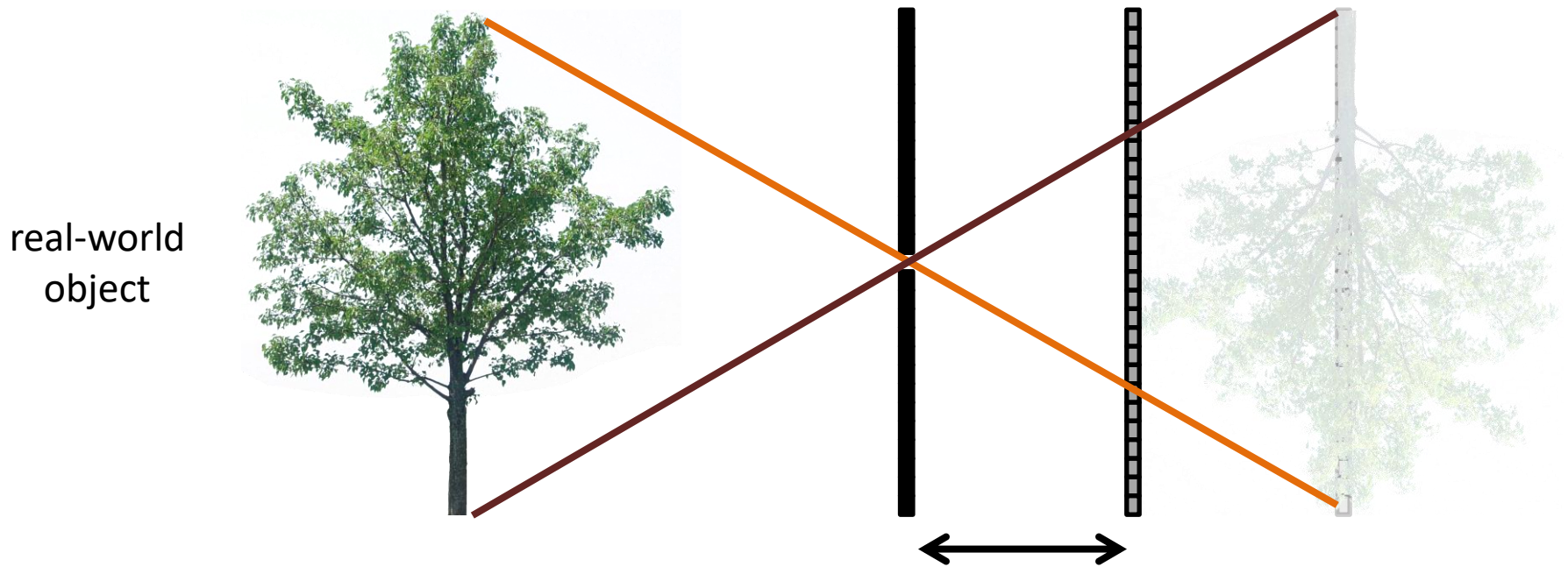


# Examples of camera obscura



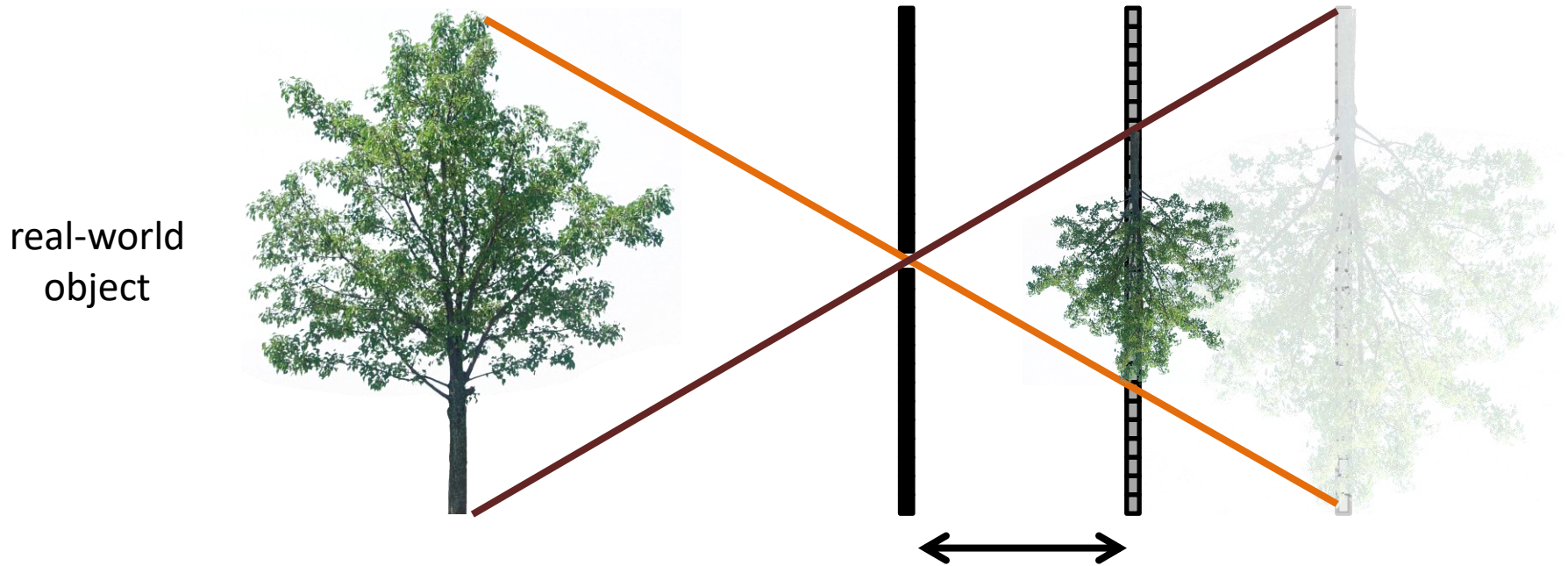
# Sensor place

- What happens as we change the place of the sensor?



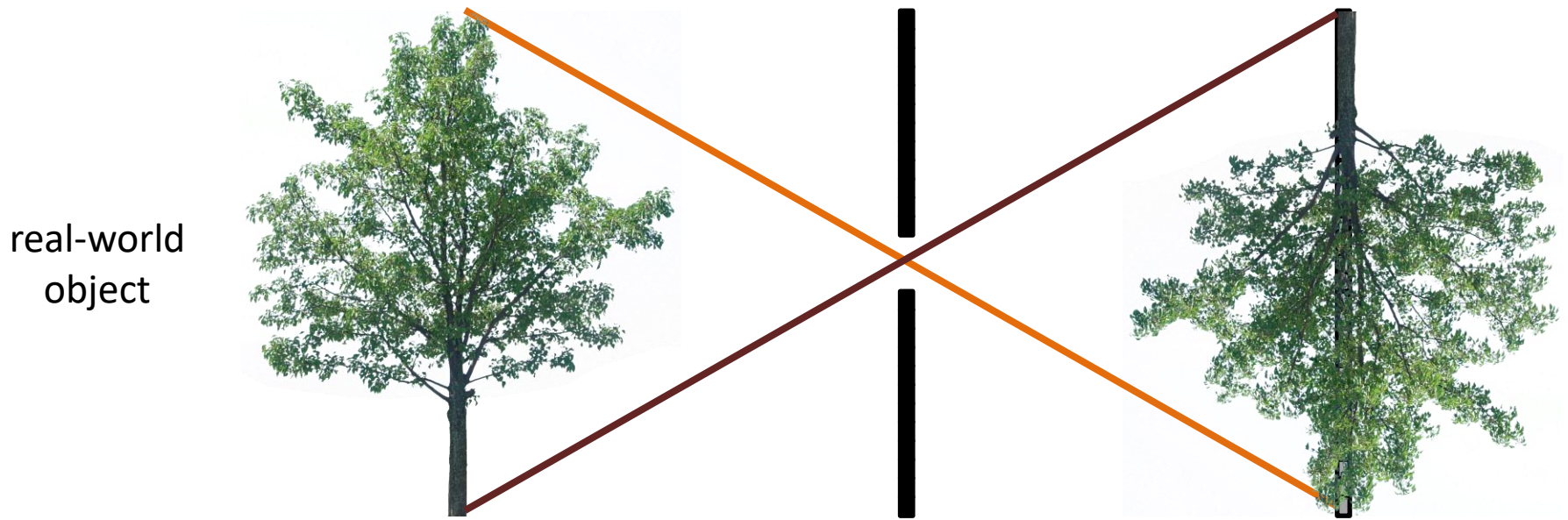
# Sensor place

- object projection is half the size.
- Changing the sensor plane distance is responsible for **zoom** in a pinhole camera.



# Pinhole size

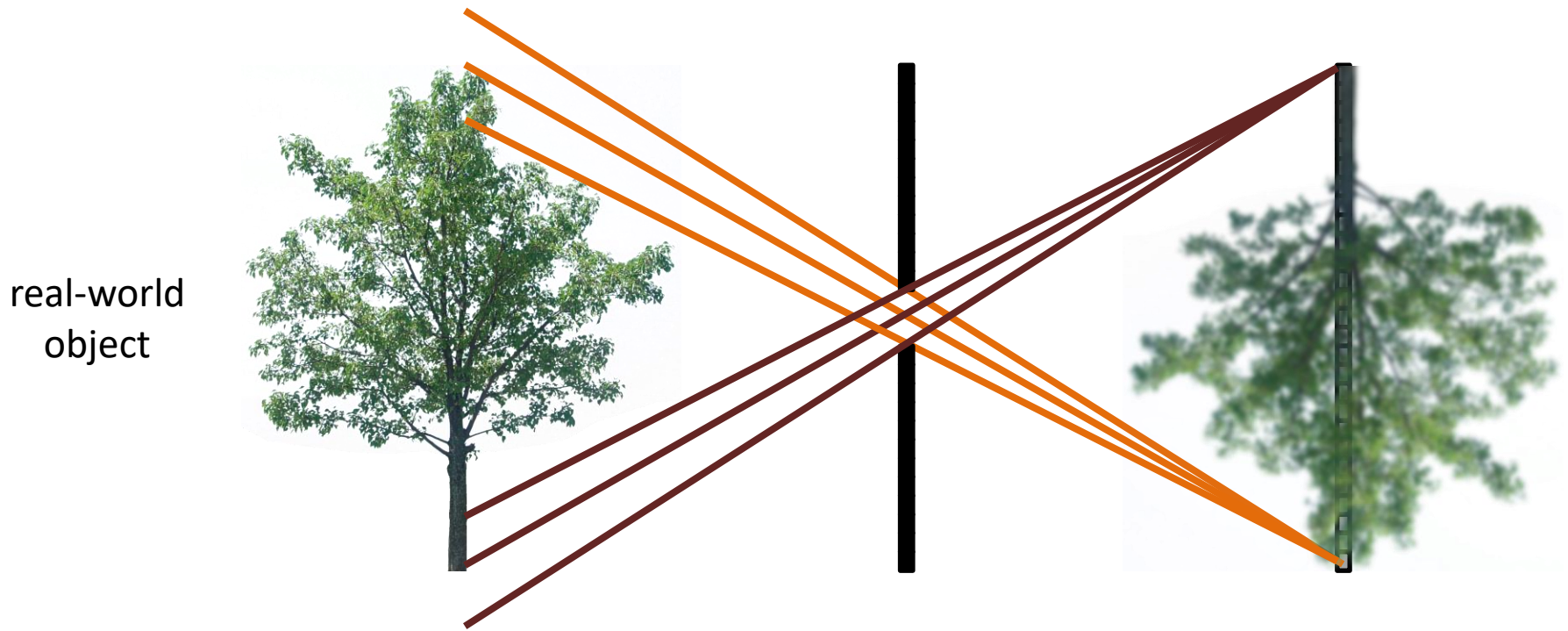
- What happens as we change the pinhole diameter?





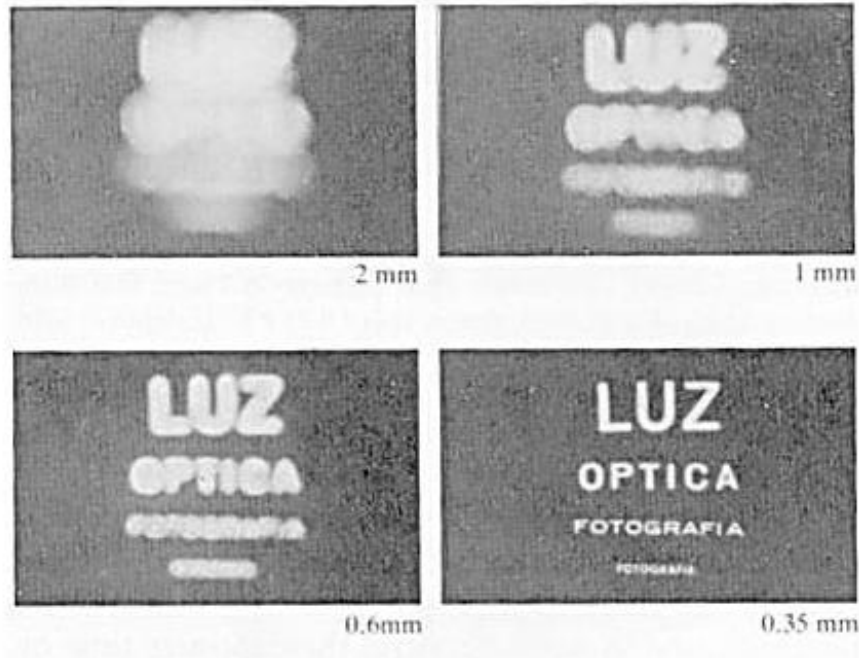
# Pinhole size

- More light from different points of the object are integrated in the same pixel (integration->sum-> blur).



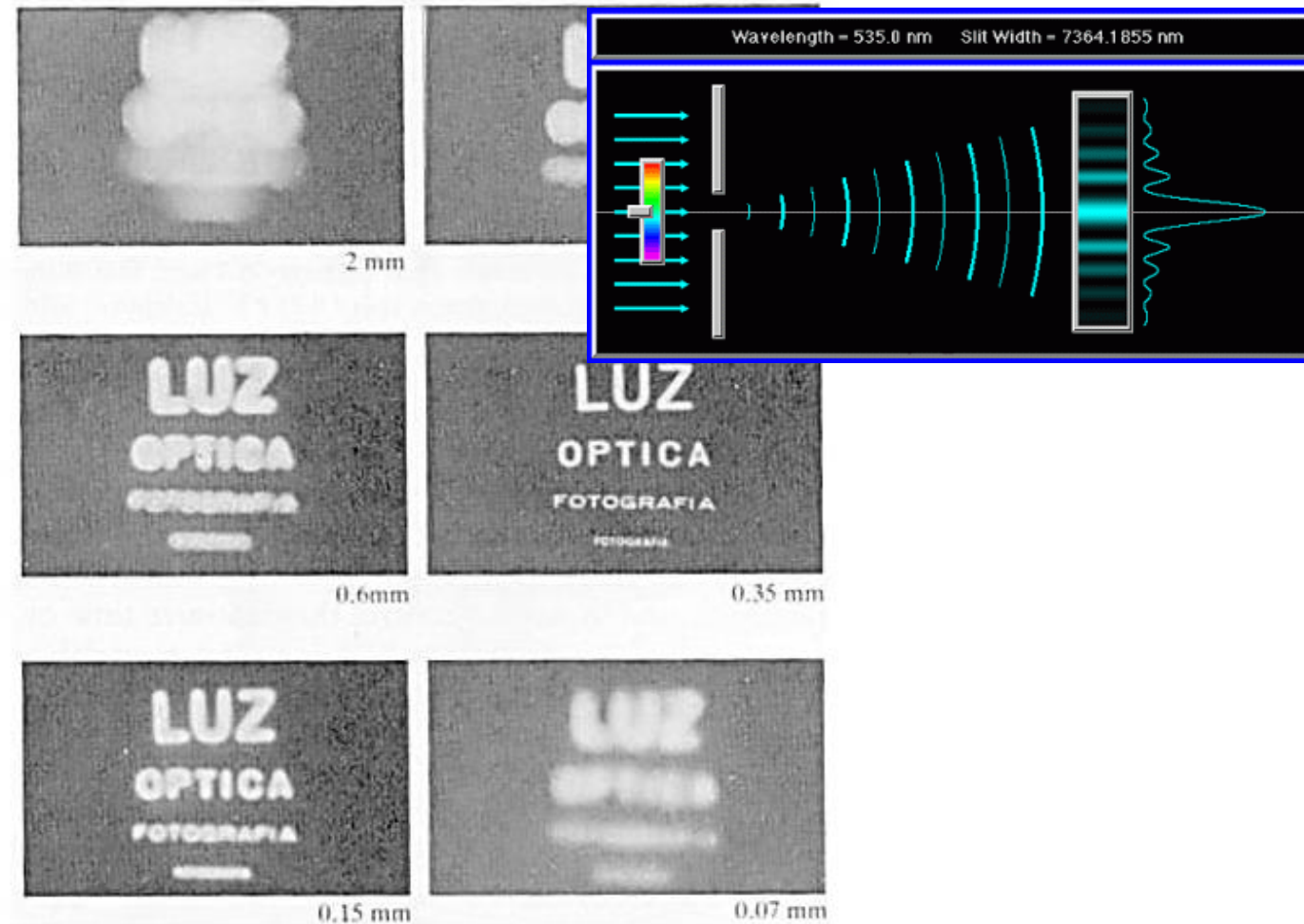
- Changing the pinhole size is responsible for **focus** in a pinhole camera.

# Side note: Pinhole size example



- Is smaller pinhole means sharper image?

# Side note: Pinhole size example



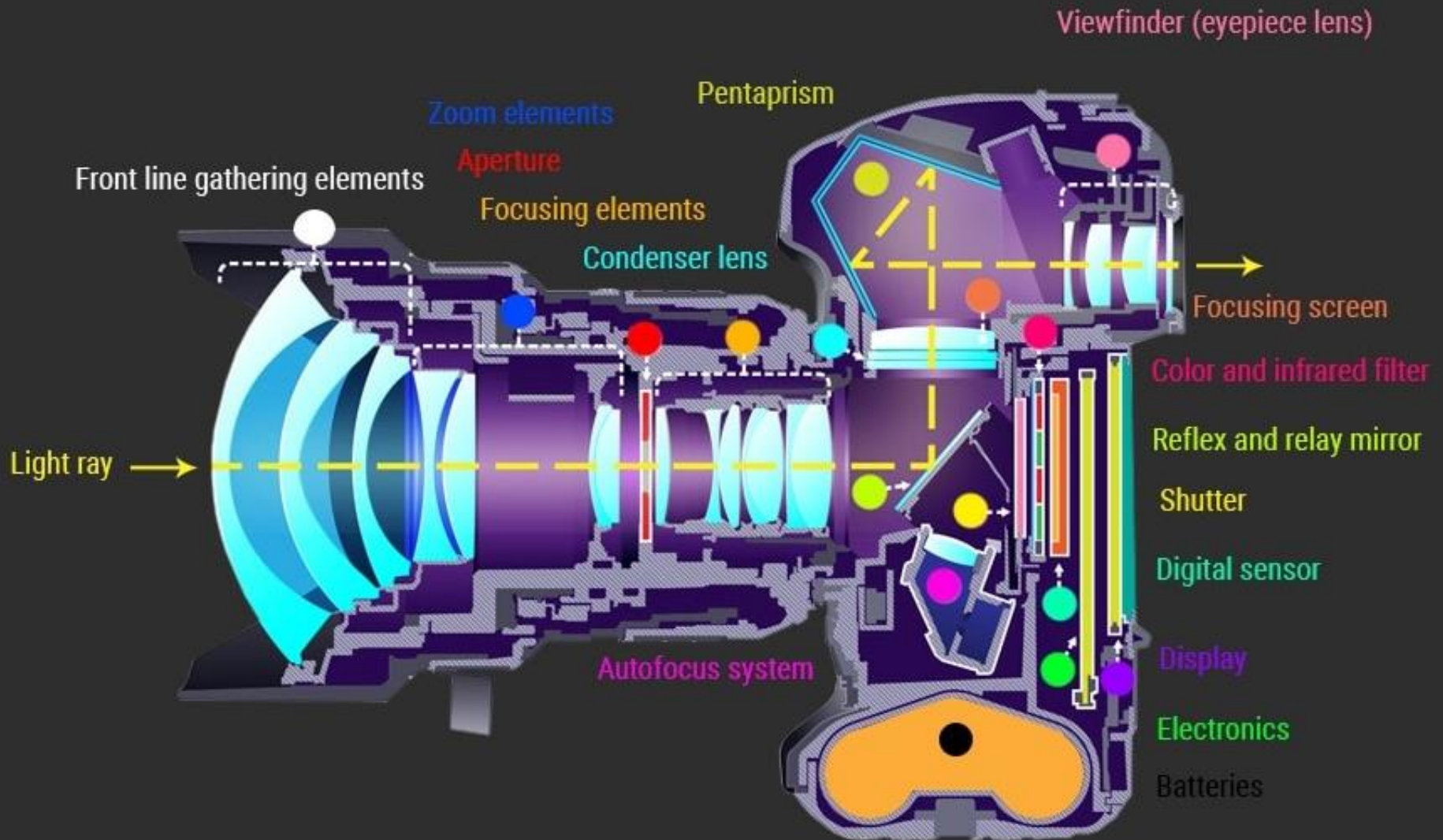
- No! due to diffraction (Wave–particle duality of photons).

# Contents

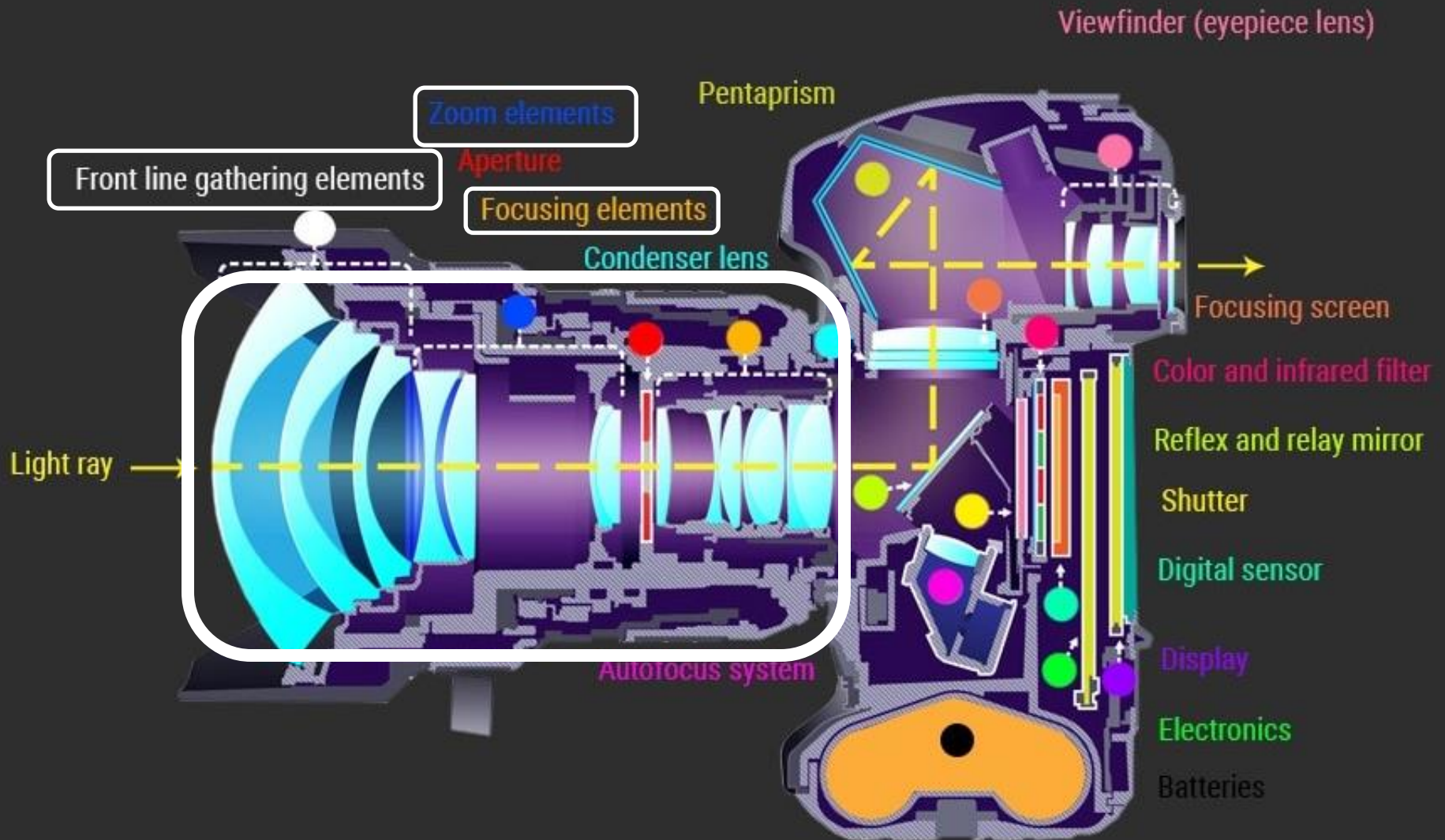
- BRDF
- Pinhole camera
- **Digital camera**
- The human eye



# Digital single-lens reflex camera (DSLR)

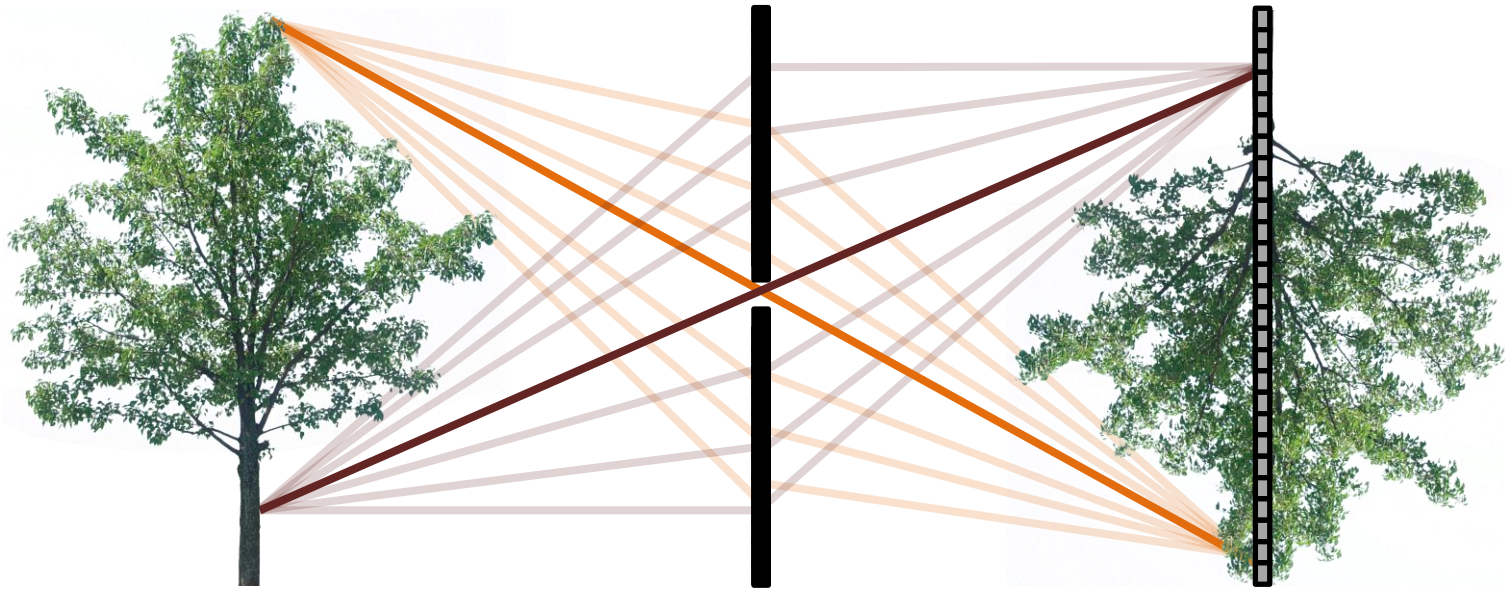


# Digital single-lens reflex camera (DSLR)



# Pinhole to lens

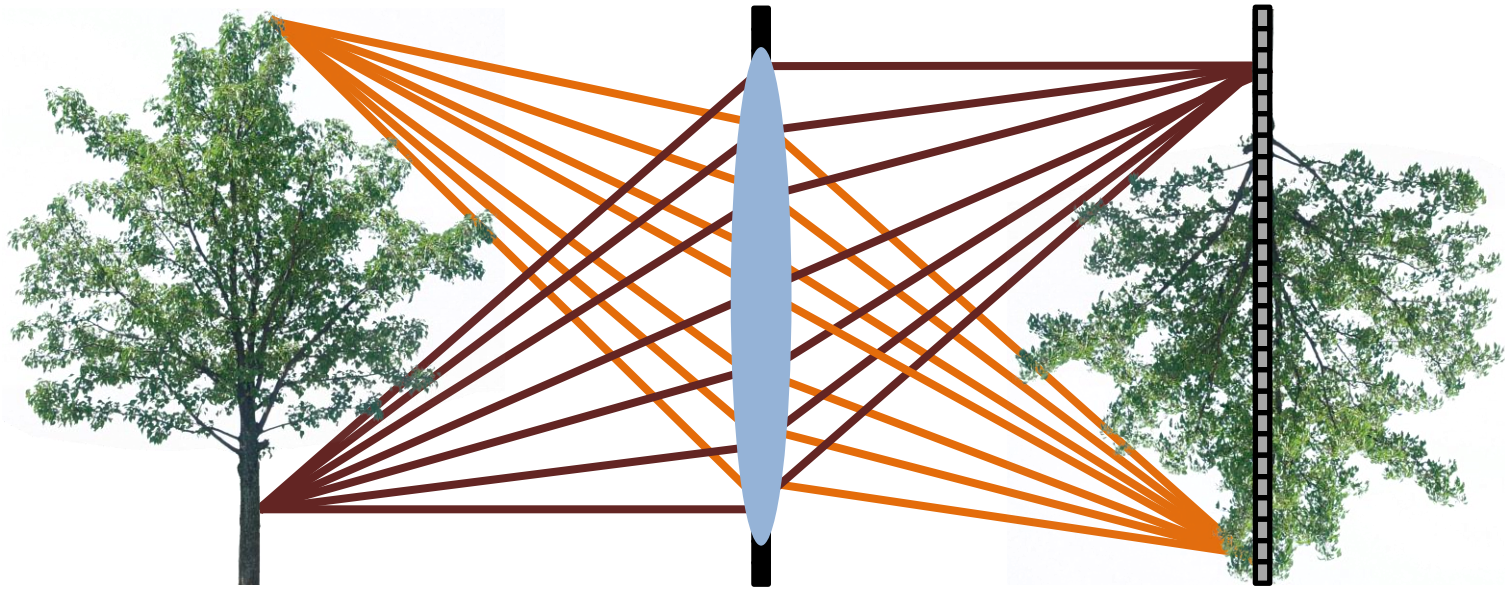
- All new cameras has lenses in them, what about it?





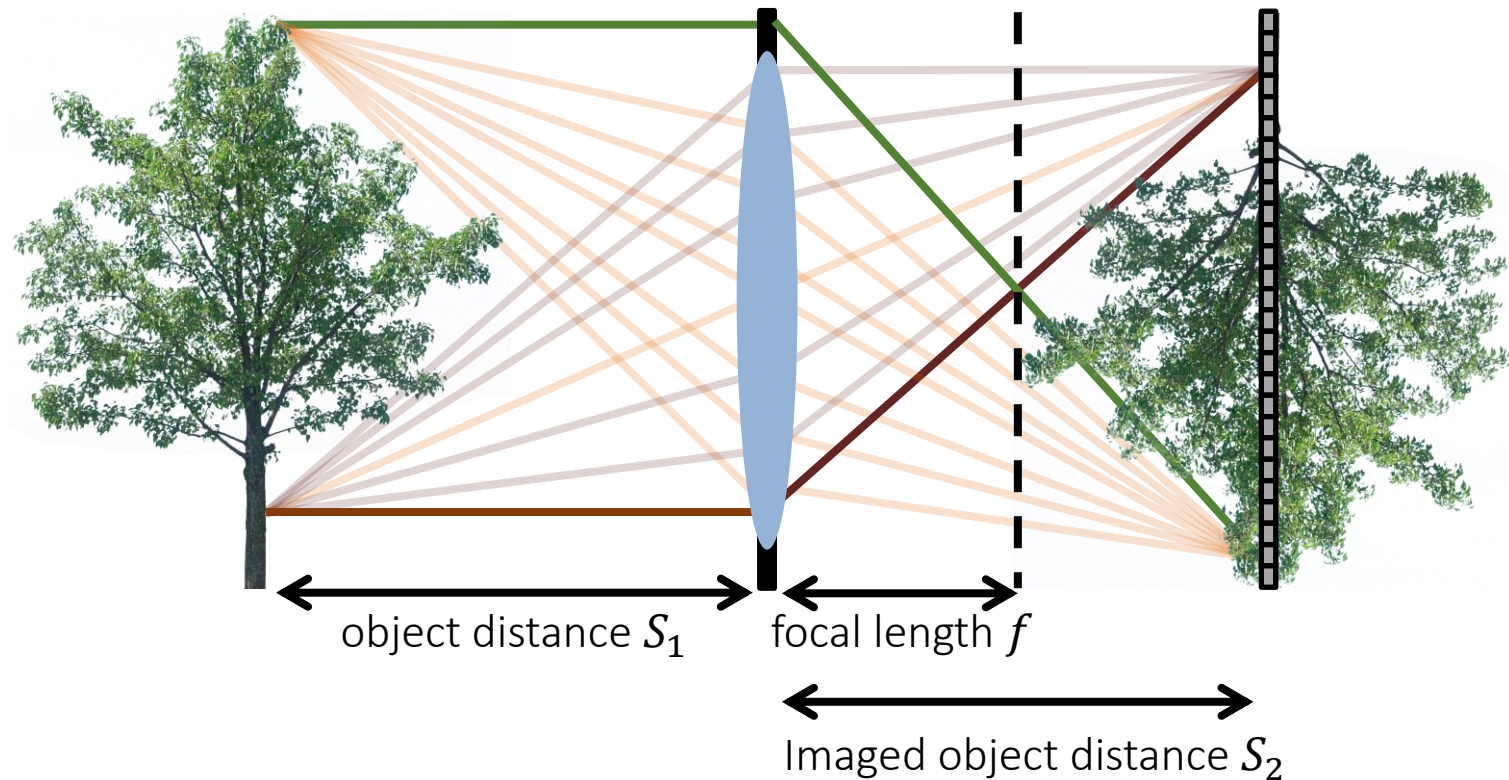
# The lens camera

- The lens can replace the pinhole, while giving the advantage of **more input light**.



# Focal length

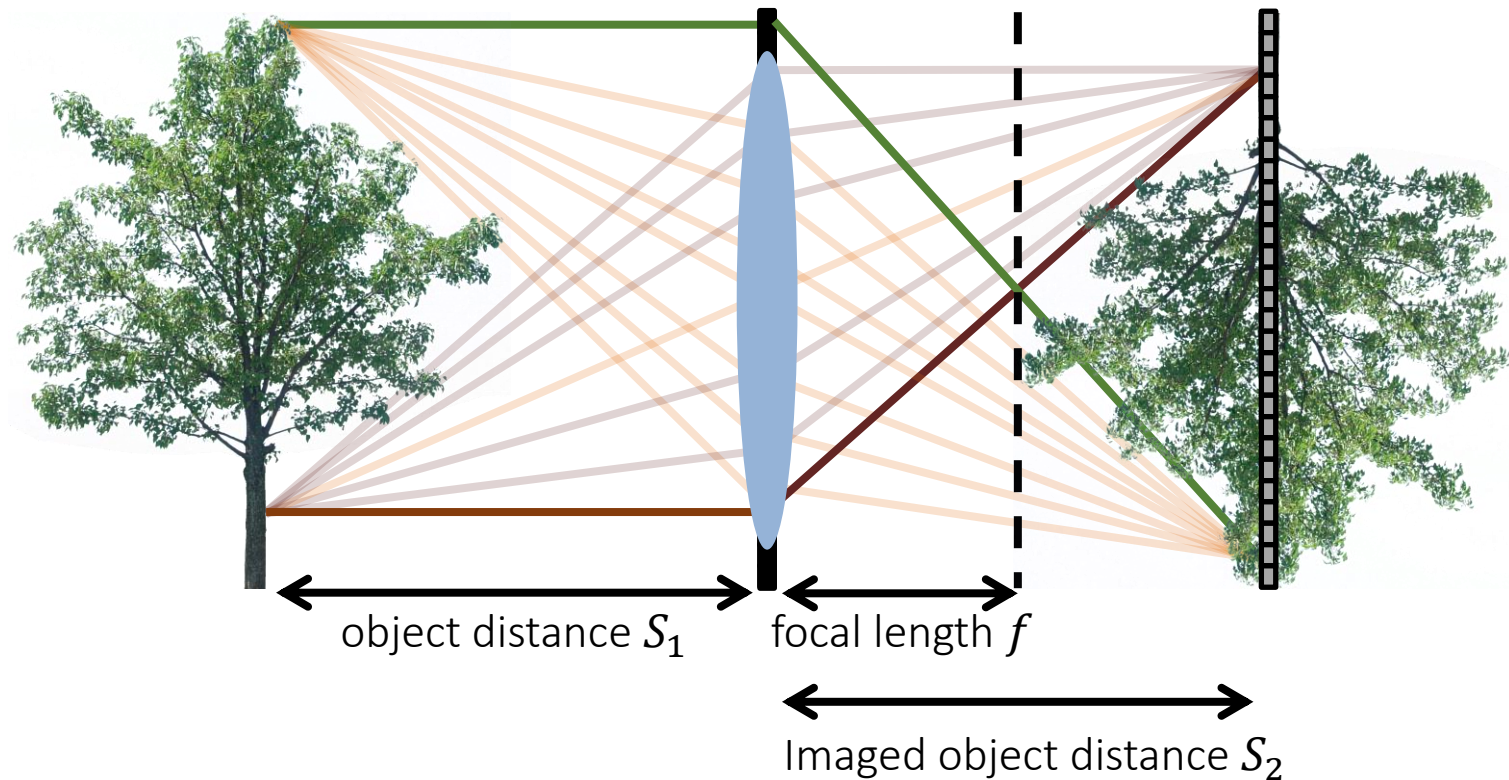
- In a lens camera, **focal length  $f$**  is the distance of intersecting parallel rays going through the lens.



# Focal length

- A known equation regarding lens is the **thin lens**

**equation:**  $\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$



# Side note: thin lens approximation

- Thin lens equation:  $\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f}$
- The thin lens equation assumes thin lens, which is not common for most cameras. Apparently, this is still relatively correct for complex lenses.
- From here on we will assume we are dealing with thin lenses—known as the **thin lens approximation**.



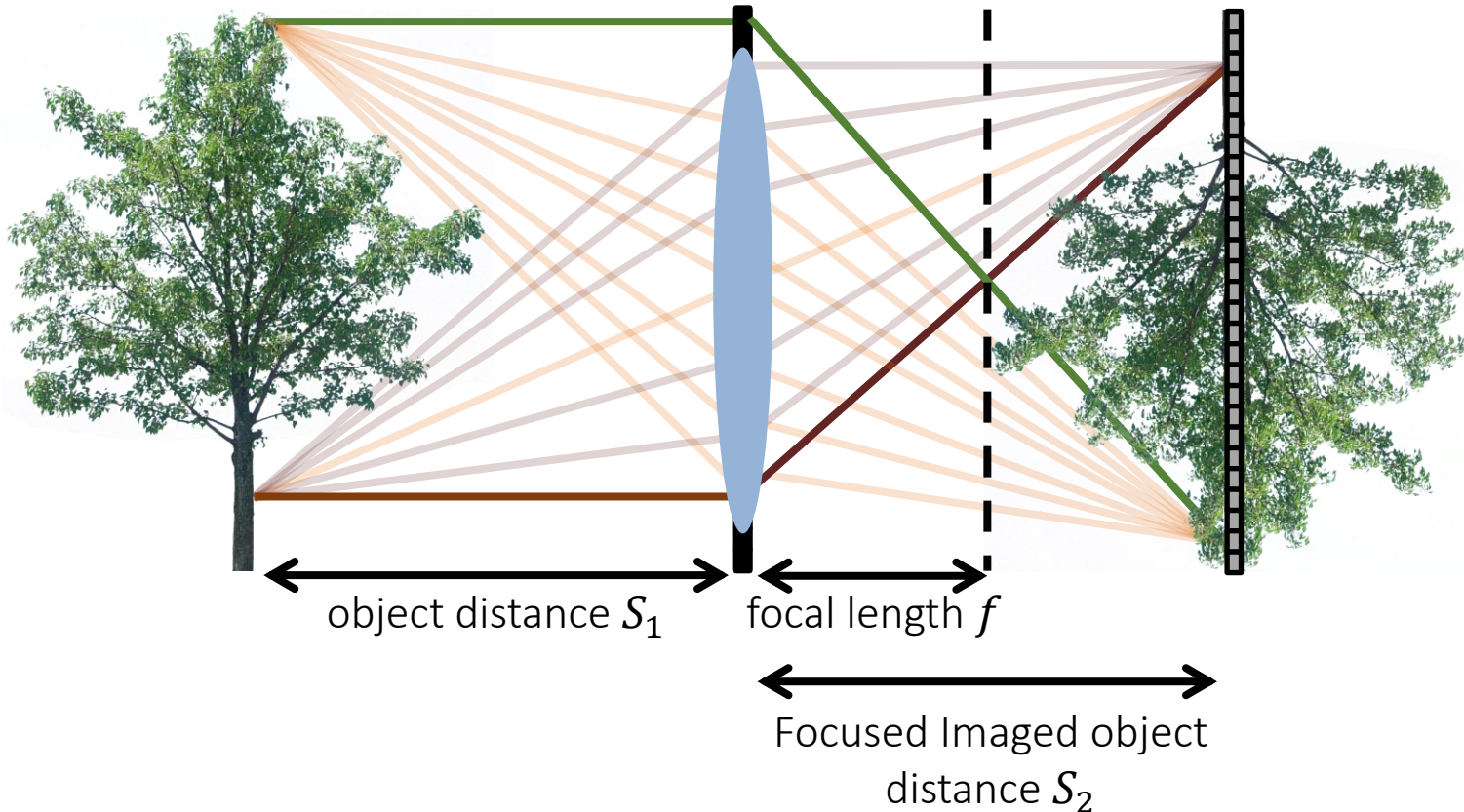
# Focal length

- Thin lens equation:  $\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f}$
- The focal length  $f$  is determined by the curvature of the lens itself (**Lensmaker's equation**- out of scope).
- What happens when a sensor is placed at  $S_2$ ? What happens when it isn't?



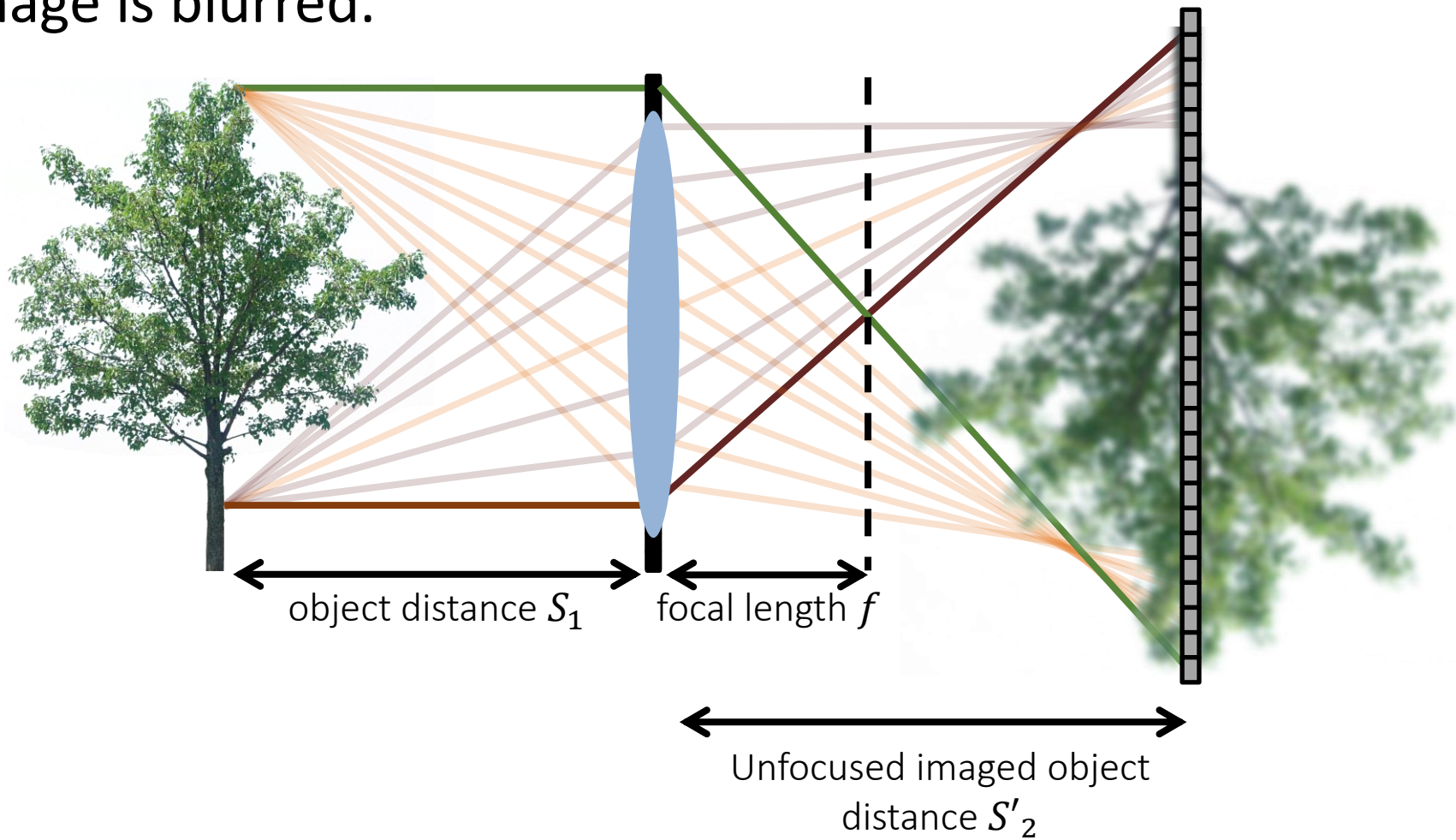
# Focal length

- Thin lens equation:  $\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$
- As already seen, when a sensor is placed at  $S_2$  the resulted image is focused.



# Focal length

- Thin lens equation:  $\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f}$
- when the sensor doesn't respect the lens equation, the image is blurred.

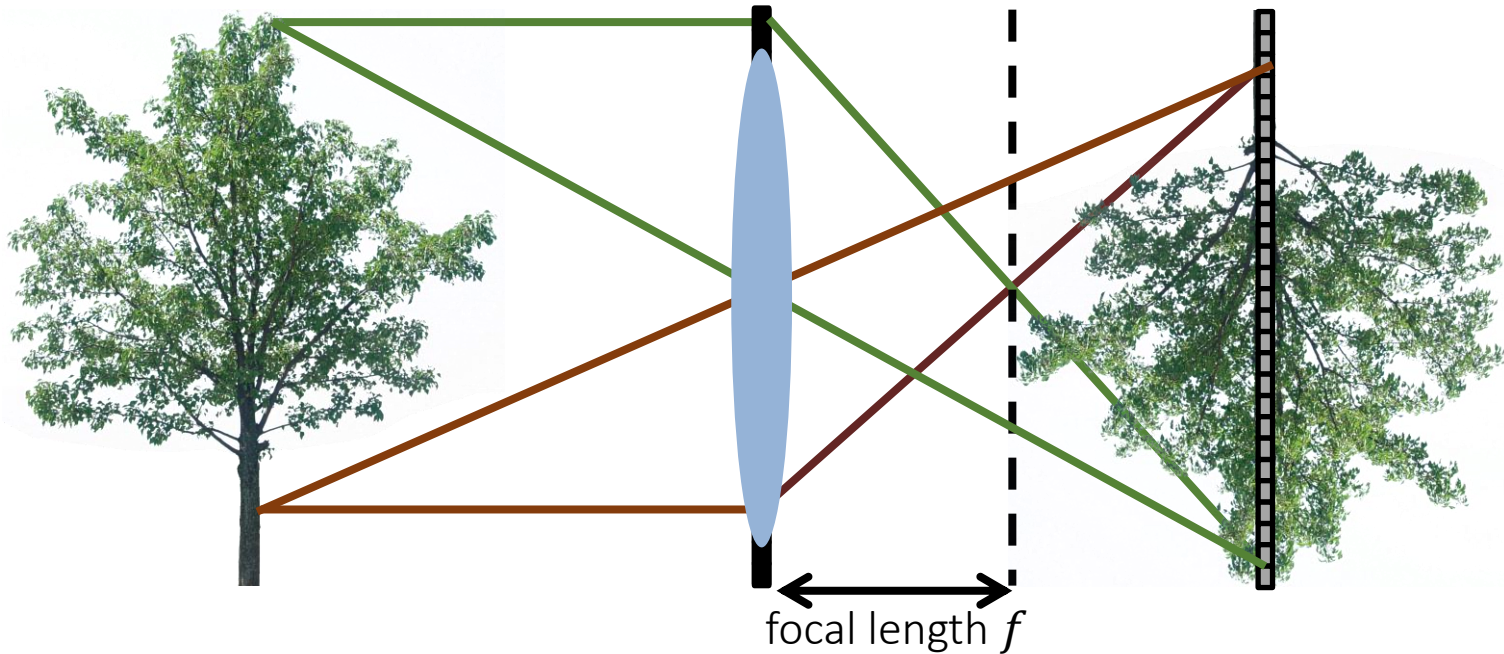


# Focus

- Changing the distance between the sensor and a given lens to make the object sharper.
- Changing the focus in a complex lens setup essentially changes the place of the approximated thin lens relative to the sensor.

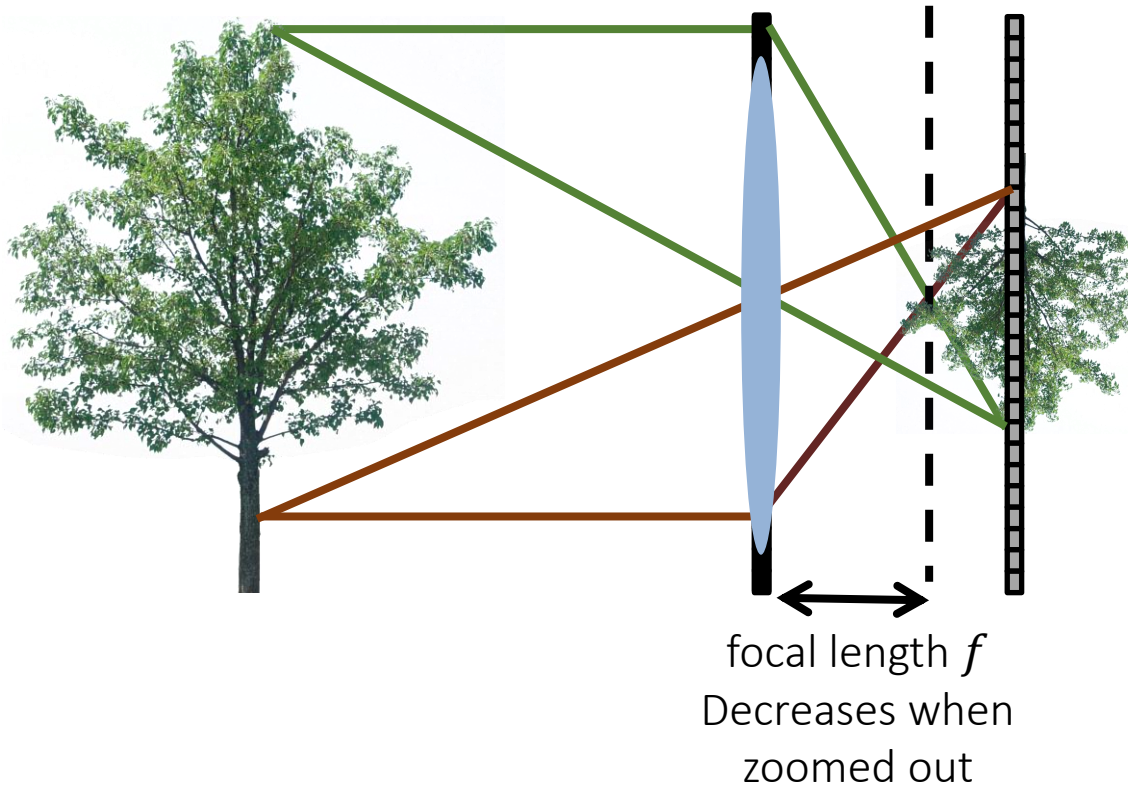
# Zoom

- Changing the relation between lenses in complex lens setup effectively changes the focal length of the thin lens approximation and by that changing the size of the perceived object.



# Zoom

- Changing the relation between lenses in complex lens setup effectively changes the focal length of the thin lens approximation and by that changing the size of the perceived object.

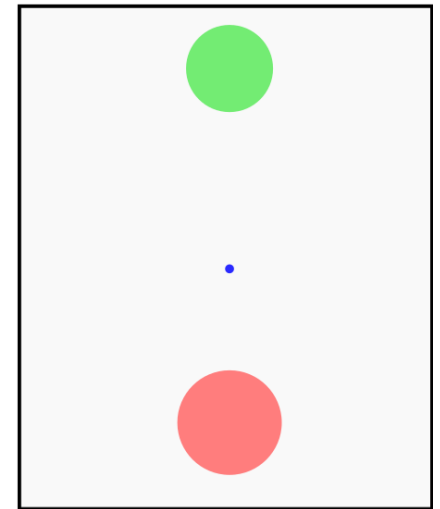
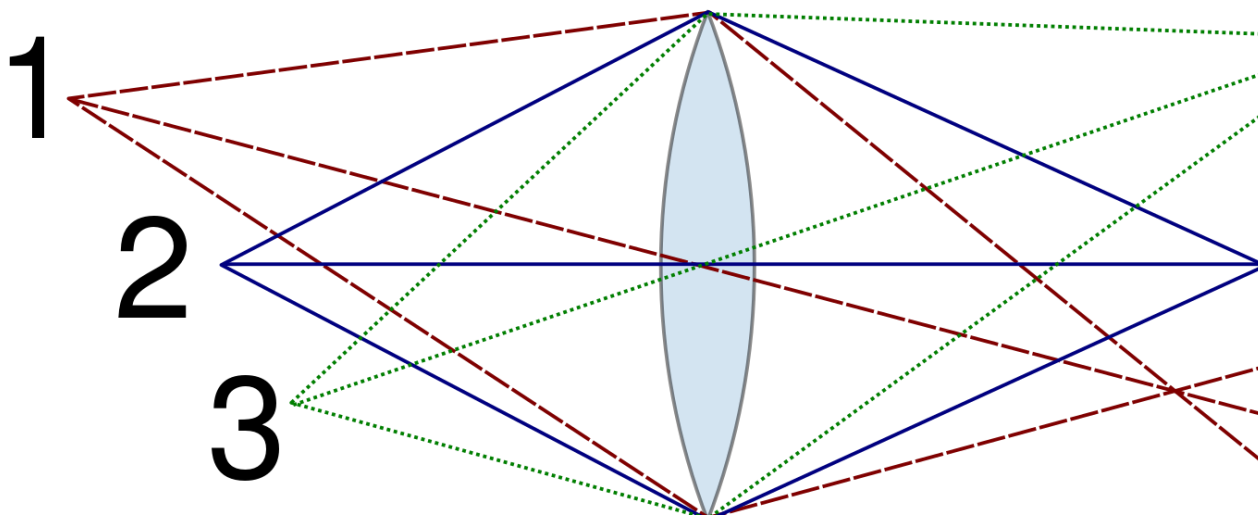




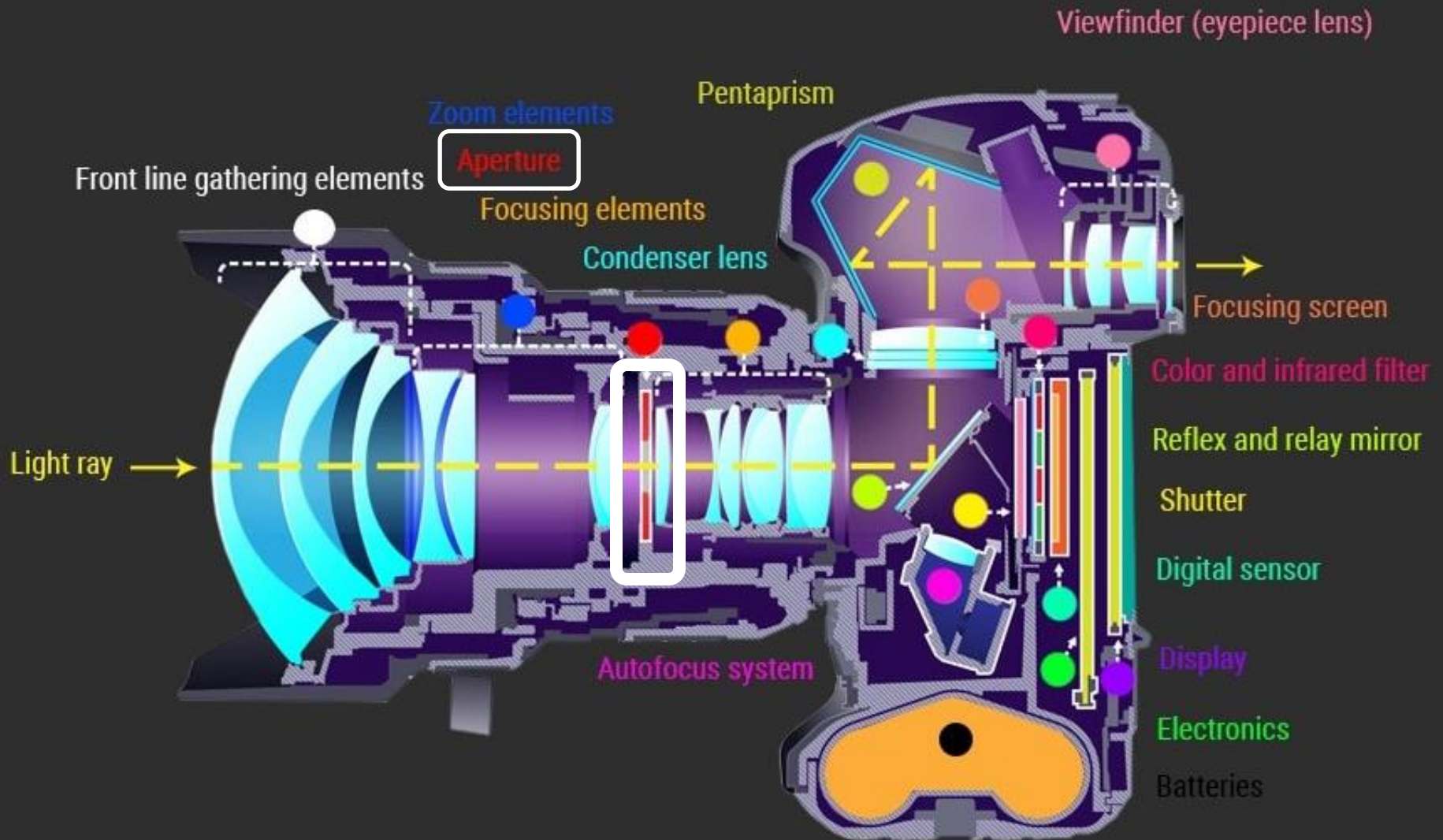
# Depth of field (DOF)

- **Depth of field** is the distance between the nearest and the farthest objects that are in acceptably sharp focus in an image.
  - “acceptably sharp” is human determined and is measured by the spot size that a point light is out of focus (called **circle of confusion**).

... infinity, the size of the circle of confusion for that aperture. The scales on a lens barrel are marked with the hyperfocal distance opposite the aperture you are using. If you use the hyperfocal distance, the depth of field will extend to infinity. For a camera with a hyperfocal focus at 18 feet,

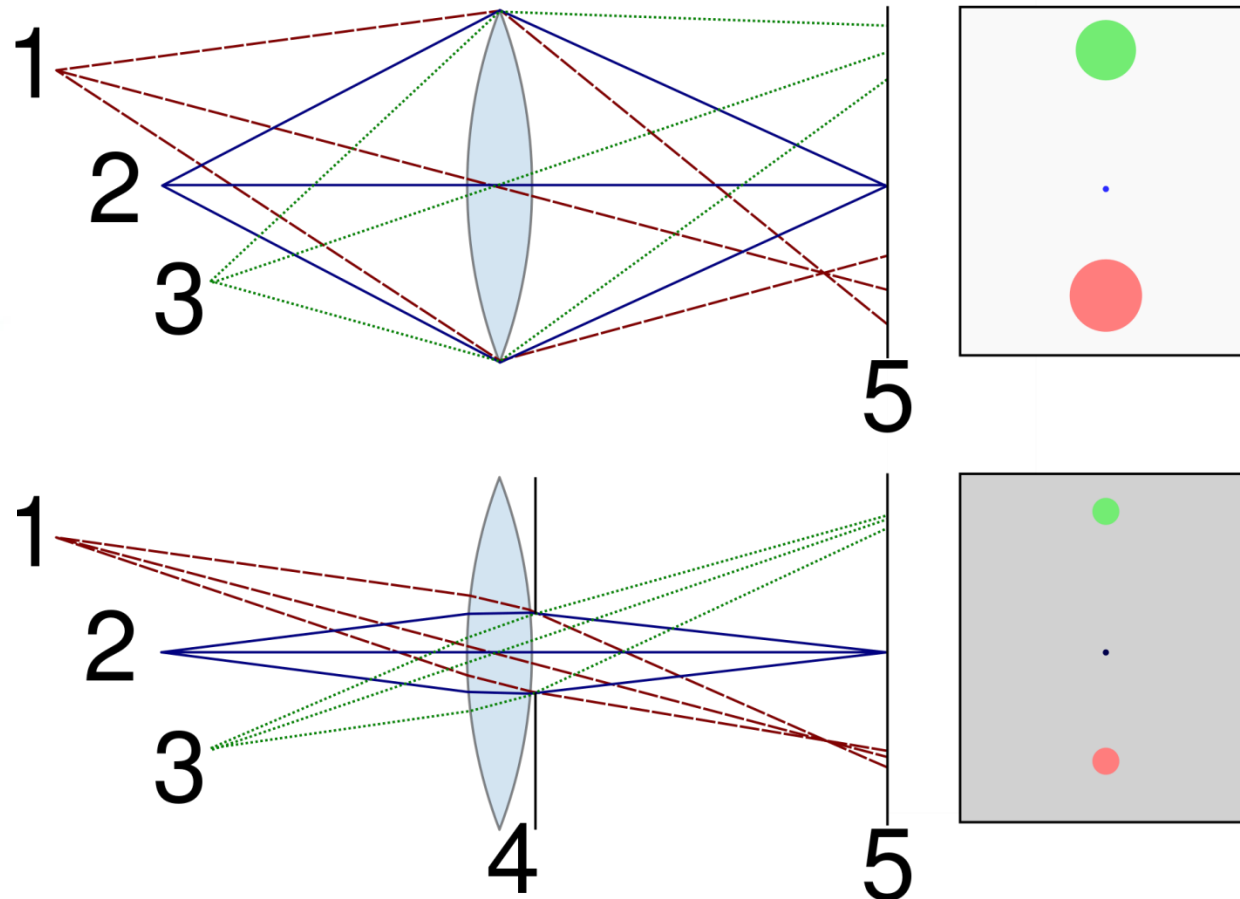


# Digital single-lens reflex camera (DSLR)



# Aperture

- An **aperture** is a hole or an opening through which light travels. The aperture helps enlarging the DOF, but also inputs less light, making the image darker.





# Side note: aperture blur

- Changing the default shape of the aperture will give different shapes to “circle of confusion”.

*f/1.4*



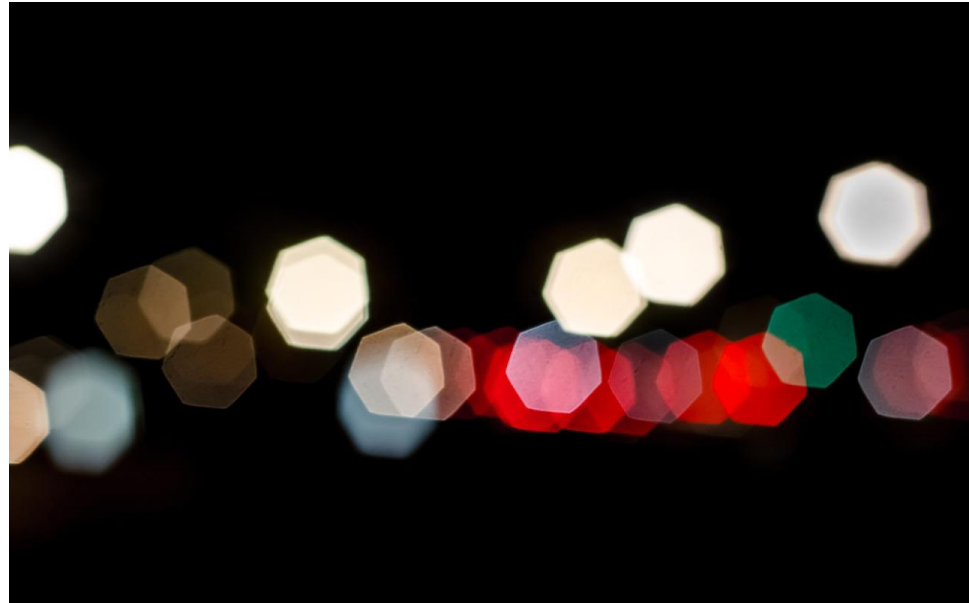
*f/2.0*



*f/4.0*

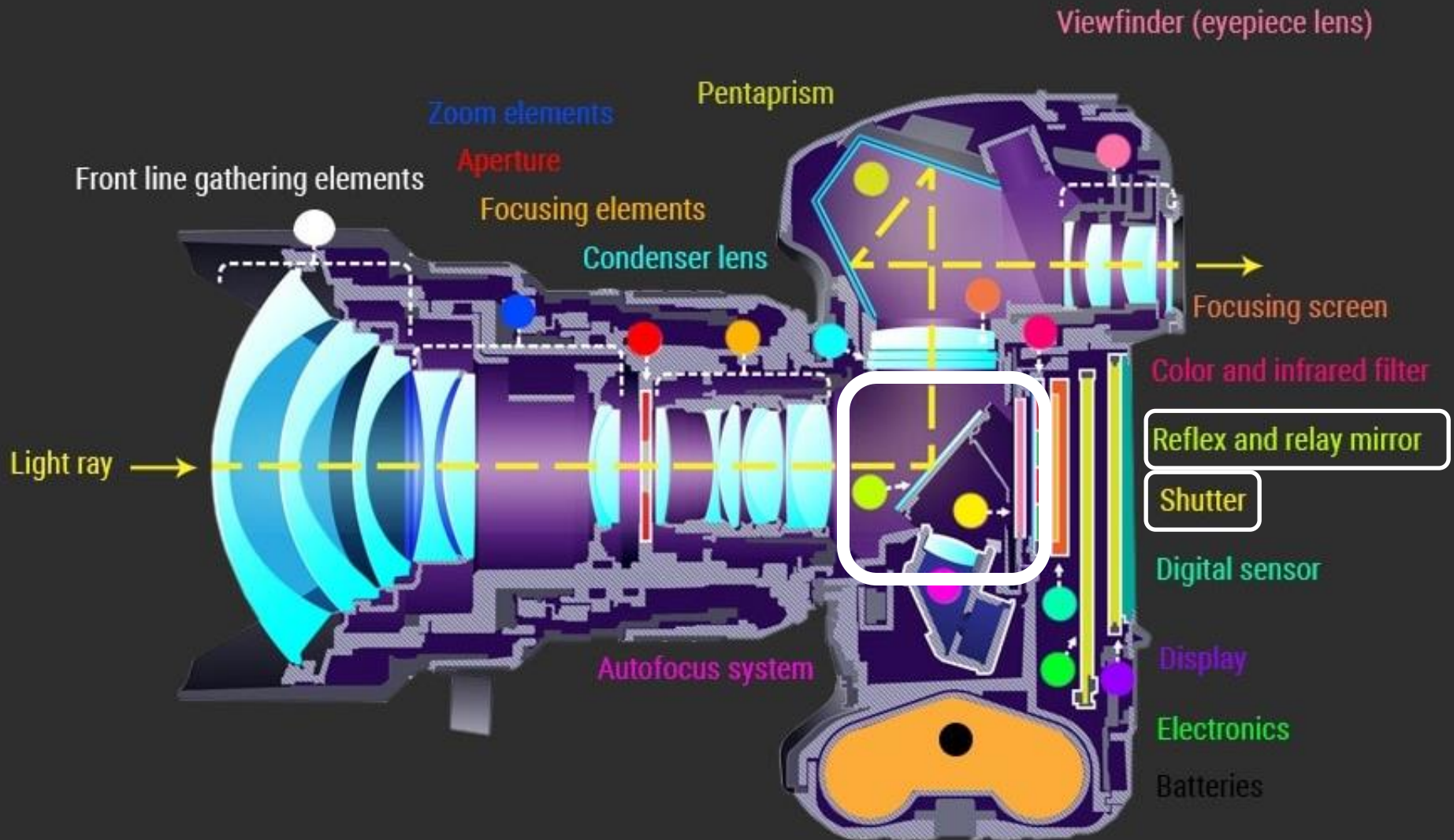


*f/5.6*





# Digital single-lens reflex camera (DSLR)



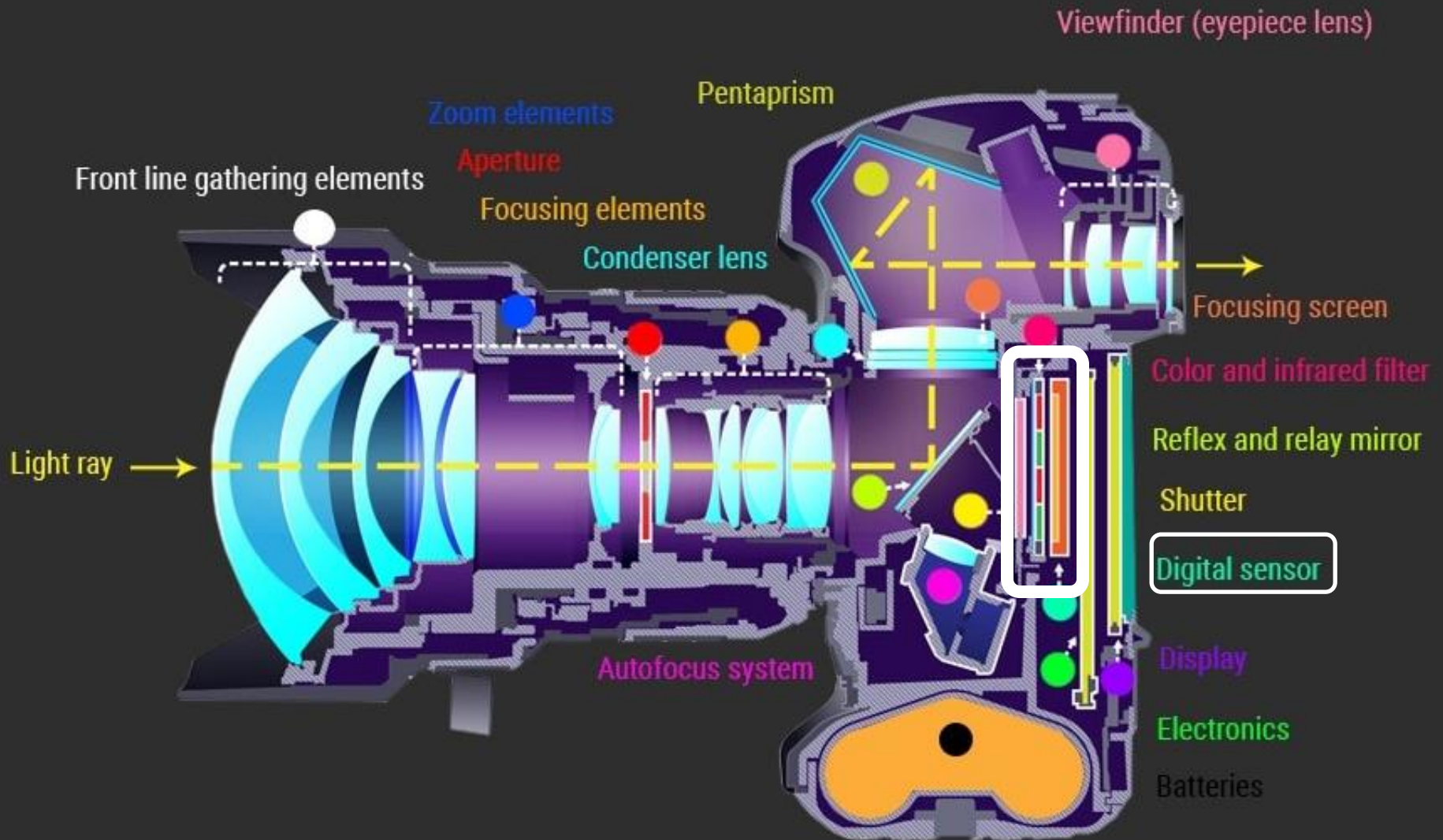
# shutter

- A common way to make the image lighter is to increase **exposure time** (also called “**shutter speed**”, although measurements units are seconds).
- Larger exposure time can also lead to image blur if camera is not stable during capture
- <https://www.youtube.com/watch?v=ptfSW4eW25g>  
(shutter and reflex mirror operation slow-motion)



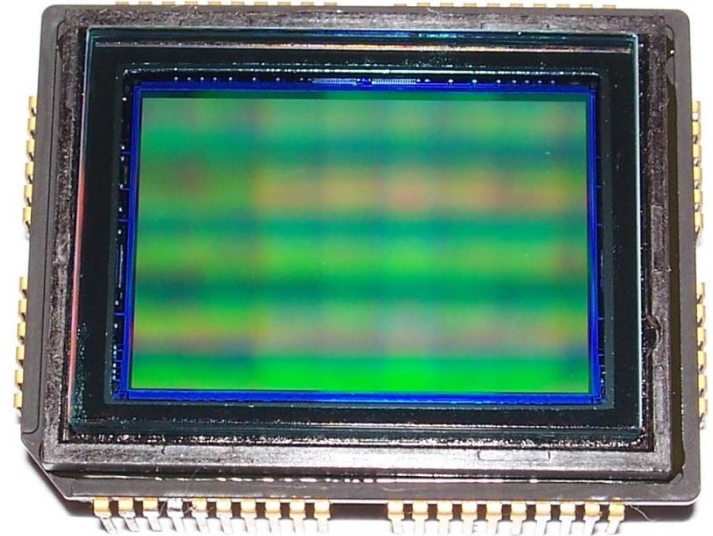


# Digital single-lens reflex camera (DSLR)



# Photosensors

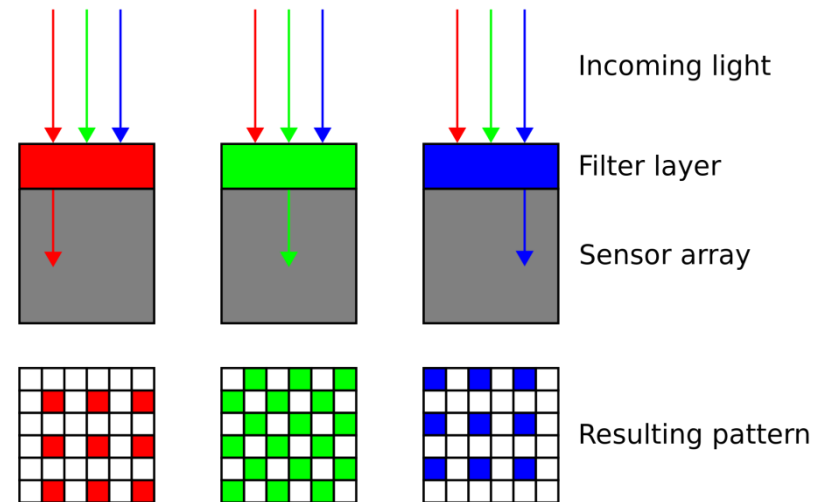
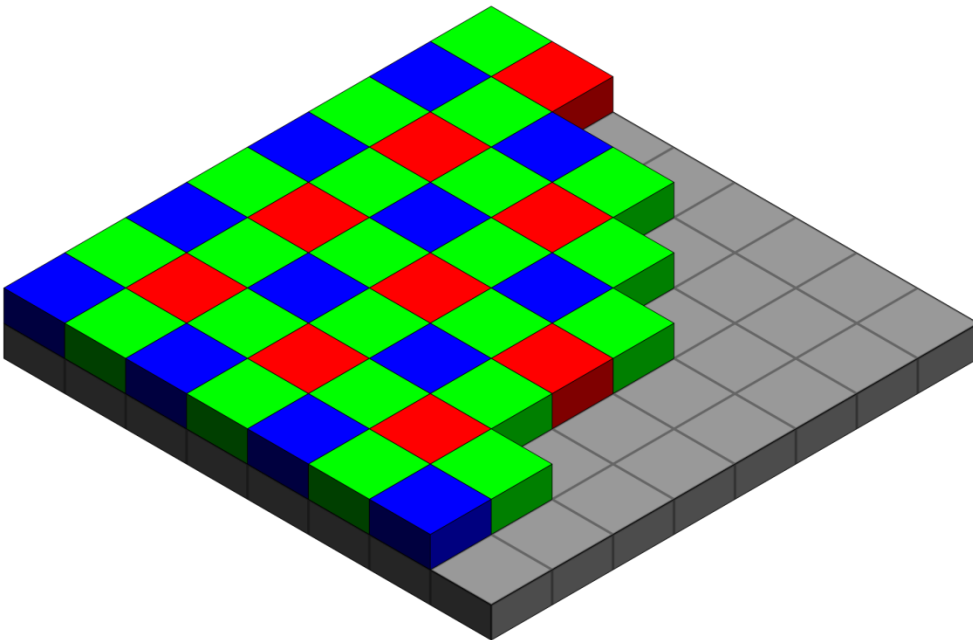
- Sensor that transforms different input light intensities to different valued electrical charges.
  - Each pixel in the resulting color image comes from at least 3 capacitors in a 2D array which makes the sensor (R, G and B).
- Two types of such sensors are called **CCD** (charge-coupled device) or **CMOC** (complementary metal-oxide-semiconductors).





# Bayer filter

- On top of each photosensor is a Bayer filter which filters the input light to one of the 3 primary colors R,G & B.
- This filter has twice as much green elements compared to the others, to mimic the human eye physiology.

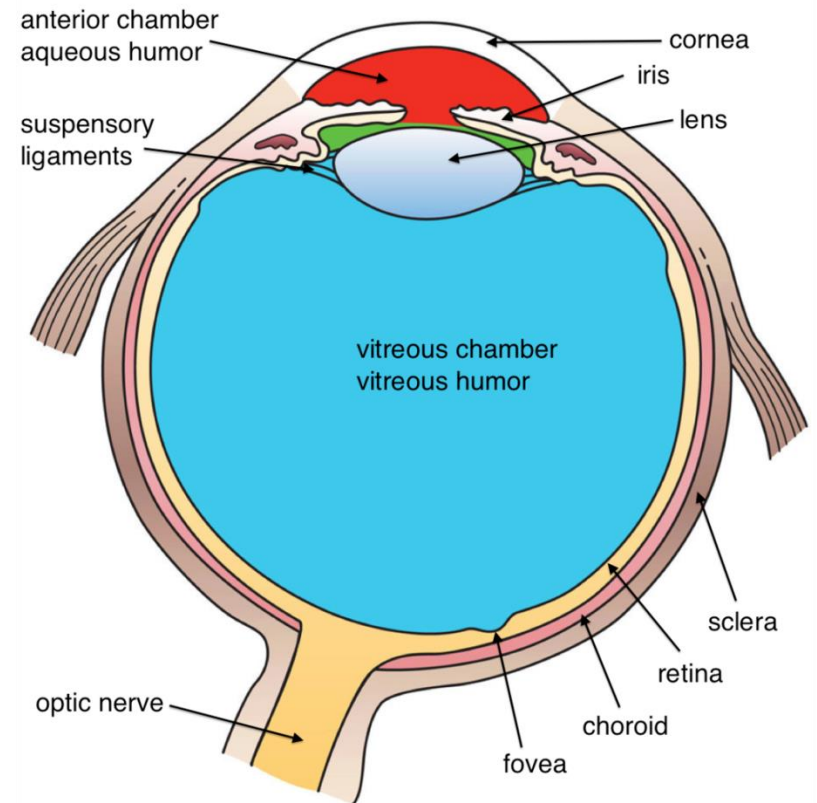


# Contents

- BRDF
- Pinhole camera
- Digital camera
- **The human eye**

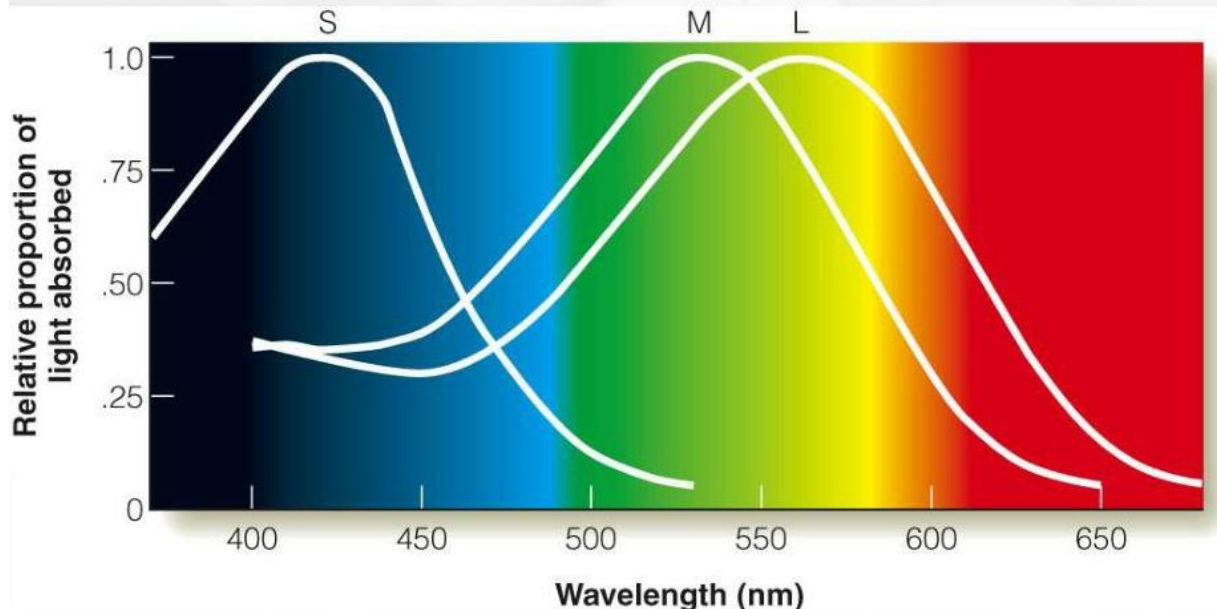
# Human color perception

- The modern camera tries to mimic the human eye:
  - Iris = aperture.
  - Lens.
  - Retina = photosensor.



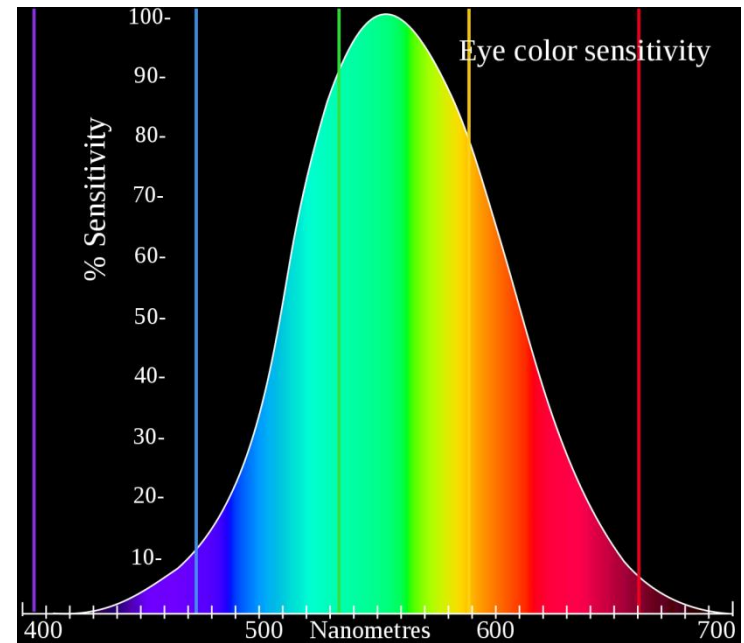
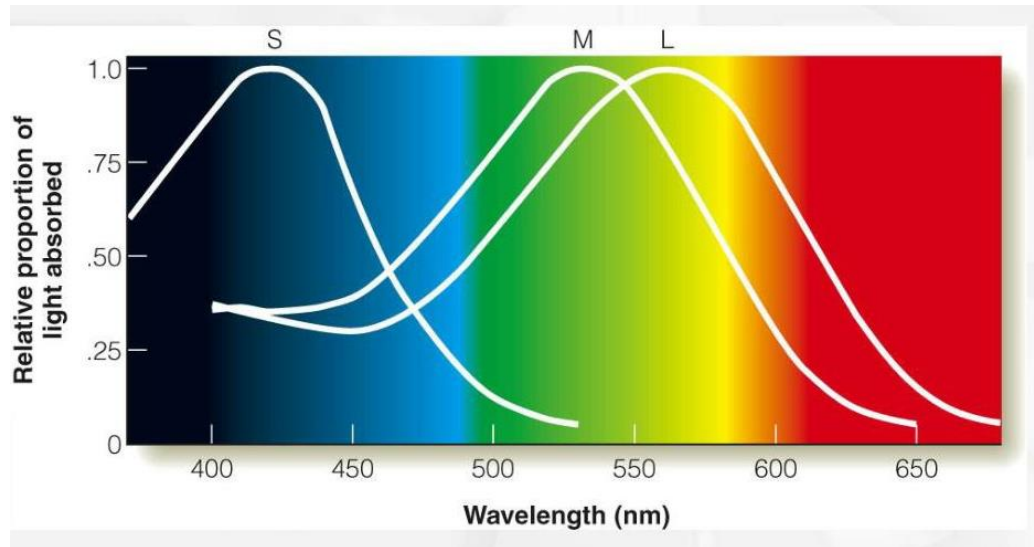
# Human color perception

- In the retina there are two types of photoreceptors:
  - Rods: responsible for low-light vision – maximum sensitivity at the green region of 500nm. Rods are **not** responsible for color vision though.
  - Cones: responsible for color vision. 3 types (short, medium & long), each has maximum sensitive for different colors.



# Human color perception

- The eye color sensitivity (Luminosity function) was defined by experiments- one result from those experiments was that people are more sensitive to the green color- can be seen by summation over the 3 stimuli of 3 kinds of cones.
- This is way Bayer filter has double the greens!



# Human color perception

- A lot more on color theory and perception in this short course (a link to the first class):
  - [https://www.youtube.com/watch?v=iDsrzKDB\\_tA](https://www.youtube.com/watch?v=iDsrzKDB_tA)